



SCIENTIFIC BULLETIN

PHYSICAL AND MATHEMATICAL RESEARCH

ILMIY XABARNOMA

FIZIKA-MATEMATIKA TADQIQOTLARI

2025
VOLUME 7
ISSUE 3

ЗАХИРИДДИН
МУҲАММАД БОБУР
НОМИДАГИ АНДИЖОН
ДАВЛАТ УНИВЕРСИТЕТИ

ANDIJAN STATE
UNIVERSITY NAMED
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MUKHAMMAD BABUR

ИЛМИЙ ХАБАРНОМА

Физика-математика
тадқиқотлари

SCIENTIFIC BULLETIN

Physical and Mathematical
Research

2025/№3(7)

Андижон
2025йил

Муассис

Захириддин Мухаммад Бобур номидаги
Андижон давлат университети

ИЛМИЙ ХАБАРНОМА. ФИЗИКА-МАТЕМАТИКА ТАДҚИҚОТЛАРИ

Журнал бир йилда 4 марта чоп этилади.

Андижон вилояти ахборот ва оммавий
коммуникациялар бошқармаси томонидан
2019-йил 26 декабрда
0452 рақам билан рўйхатга олинган.

Нашр индекси: 344

Нашр учун масъул:
Н.Й. Юнусалиев

Босишга рухсат этилди: 27.12.2019.
Қоғоз бичими: 60x81 1/8 Босма табоғи: 13,5
Офсет босма. Офсет қоғози.
Адади: 110 дона.
Баҳоси келишилган нархда.
Буюртма №: 165.
“Муҳаррир” нашриёти матбаа бўлимида чоп
этилди.
Тошкент шаҳри, Сўғалли ота кўчаси 7-уй

Таҳририят манзили:

170100, Андижон шаҳри, Университет
кўчаси, 129. Телефон: +998911602043.
Факс: (374) 223-88-30

E-mail: adu_xabarnoma@mail.ru Расмий сайт:
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ИНФОРМАТИКА

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РЕДАКТОР

Юнусалиев Нуритдин Юнусали угли – PhD по физико-математическим наукам, Андижанский государственный университет.

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UDC 538.9:621.382.2

Gamma nurlanishlar ta'sirida $n\text{-GaAs-}p\text{-(ZnSe)}_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$ geterotuzilmalarning fotosezgirlik spektridagi o'zgarishlar tahlili

S. Zaynabidinov¹, A.Y. Boboyev¹, Sh.X. Yo'Ichiyev², I.M.Soliyev²

¹Z.M.Bobur nomidagi Andijon davlat universiteti, 1700100, Andijon, O'zbekiston

²Andijon davlat pedagogika instituti, Andijon, O'zbekiston

E-mail: igboljonsoliyev45@gmail.com

2025-yil. 2-avgustda berilgan; 2025 yil. 2-noyabr chop etish uchun qabul qilingan

Annotatsiya: Ushbu maqolada $n\text{-GaAs-}p\text{-(ZnSe)}_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$ geterotuzilmalarning gamma nurlanishlar ta'siridagi fotosezgirlik spektrlari tahlil qilingan. Epitaksial plyonkalar suyuq fazali epitaksiya usuli yordamida vismut hosil qiluvchi eritmadan GaAs tagliklariga o'stirildi. Tajribalar natijasida gamma nurlanishlardan oldin va keyin geterotuzilmalarning fotosezgirlik spektrlarida sezilarli o'zgarishlar qayd etildi. Xususan, 105–106 rad gamma nurlanish ta'sirida fotosezgirlik cho'qqilarining qisqa to'lqinlar tomon siljishi, spektral kengayishlar hamda yangi mahalliyashgan energetik sathlarning hosil bo'lishi aniqlandi. Ushbu o'zgarishlar tuzilmaning sirtiy va hajmiy sohalarida chuqur sathli rekombinatsiya markazlari hosil bo'lishi bilan izohlandi. Bundan tashqari, ZnSe–GaAsBi bo'linish chegaralaridagi subkristallitlar deformatsiyasi natijasida Kvant-Konch-Shtark va Moss–Byurshteyn effektlarining birgalikdagi ta'siri qayd etildi. Olingan natijalar gamma nurlanish ta'sirida geterotuzilmalarning energetik sohalarini boshqarish imkonini beradi hamda fotosezgir qurilmalar samaradorligini oshirish uchun amaliy ahamiyatga ega.

Kalit so'zlar: $n\text{-GaAs-}p\text{-(ZnSe)}_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$, geterotuzilma, fotosezgirlik spektri, gamma nurlanish, epitaksial plyonka, rekombinatsiya markazlari, Moss–Byurshteyn effekti, Kvant-Konch-Shtark effekti.

Аннотация: В данной статье представлен анализ спектров фоточувствительности гетероструктур $n\text{-GaAs-}p\text{-(ZnSe)}_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$ под воздействием гамма-излучения. Эпитаксиальные плёнки выращены на подложках GaAs из висмутсодержащего расплава методом жидкофазной эпитаксии. Экспериментальные результаты показали существенные изменения в спектрах фоточувствительности гетероструктур до и после облучения гамма-лучами. В частности, после воздействия доз 105–106 рад наблюдались смещение максимумов фоточувствительности в коротковолновую область, расширение спектра и образование новых локализованных энергетических состояний. Эти изменения объясняются формированием глубоких уровней рекомбинационных центров в поверхностных и объёмных областях структур. Кроме того, деформация субкristаллитов на границе ZnSe–GaAsBi приводит к совместному проявлению эффектов Квантово-Конфайнед-Штарка и Мосса–Бюрштейна. Полученные результаты указывают на возможность управления энергетическими зонами гетероструктур под воздействием гамма-излучения и могут быть использованы для повышения эффективности фоточувствительных устройств.

Ключевые слова: $n\text{-GaAs-}p\text{-(ZnSe)}_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$, гетероструктура, спектр фоточувствительности, гамма-излучение, эпитаксиальная плёнка, рекомбинационные центры, эффект Мосса–Бюрштейна, эффект Квантово-Конфайнед-Штарка.

Abstract: This article presents an analysis of the photosensitivity spectra of $n\text{-GaAs-}p\text{-(ZnSe)}_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$ heterostructures under the influence of gamma irradiation. Epitaxial films were grown on GaAs substrates from a bismuth-containing melt using the liquid-phase epitaxy method. Experimental results revealed significant changes in the photosensitivity spectra of the heterostructures before and after gamma irradiation. In particular, after exposure to 105–106 rad of gamma rays, a shift of photosensitivity peaks toward shorter wavelengths, spectral broadening, and the formation of new localized energy states were observed. These changes are attributed to the generation of deep-level recombination centers in the surface and bulk regions of the structures. Additionally, deformation of subcrystallites at the ZnSe–GaAsBi interface led to the combined manifestation of the Quantum-Confined Stark and Moss–Burststein effects.

The obtained results demonstrate the possibility of controlling the energy band structure of heterostructures under gamma irradiation and can be applied to enhance the performance of photosensitive devices.

Keywords: $n\text{-GaAs-}p\text{-(ZnSe)}_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$, heterostructure, photosensitivity spectrum, gamma irradiation, epitaxial film, recombination centers, Moss–Burstein effect, Quantum-Confined Stark effect.

Kirish.

Energiyaga bo'lgan ehtiyoj hamda energetikaning rivojlanish tendensiyasi muqobil energiya manbalariga bo'lgan talablarni sezilarli darajada oshishiga olib keladi. Yorug'lik energiyasi konvertorlar yoki quyosh batareyalari eng istiqbolli, ekologik toza energiya manbalari bo'lib, organik va noorganik manbalardan farqli o'laroq, quyosh nurlanishini to'g'ridan-to'g'ri elektr energiyasiga aylantiradi [1]. Quyosh energiyasining kelajagi endi AIIIBV va AIIIVI birikmalariga asoslangan yarimo'tkazgichli qattiq qorishmalar bilan bog'liq. Yarimo'tkazgichli quyosh elementlari tomonidan energiya konvertatsiyasining eng muhim va asosiy muammolaridan biri bu energetik soha osti va usti fotonlarining yo'qolishi, shuningdek, zaryad tashuvchilarning termalizatsiyasi hisoblanadi. Shuning uchun quyosh nurlarini qabul qilishda yuqori samaradorlik ko'rsatkichlariga erishish uchun yangi yarimo'tkazgichli materiallar va fotokonvertorlarning qurilmalarmalar yaratish talab etiladi. Ushbu yo'nalishdagi eng samarali yondashuv turli xil energiyaviy sohalarga ega bo'lgan ko'p tutashuvli (kaskadli) geterotuzilmalar asosida fotovoltaiik konvertorlarga asoslangandir.

Biroq, ko'p tutashgan geterotuzilmalar sonining ko'payishi fotovoltaiik konvertorlarni olish uchun texnologik sharoitlarning murakkablashishiga olib keladi. Bundan tashqari, yuqori samarali ko'p o'tish kaskad elementlarini yaratish uchun doimiy kristalli panjaralarni tekislashni ta'minlaydigan cheklangan miqdordagi materiallar mavjud [2].

Tadqiqot metodi.

Tadqiqotlarda taglik sifatida radiusi 10 mm va qalinligi taxminan 350 mkm bo'lgan kremniy plastinalar ishlatildi. Ularning solishtirma qarshiligi $10 \Omega \cdot \text{sm}$, tok tashuvchilar konsentratsiyasi $4,4 \cdot 10^{14} \text{ sm}^{-3}$, harakatchanligi esa $1439 \text{ sm}^2/(\text{V} \cdot \text{s})$ ni tashkil etdi. Olingan epitaksial plyonkalarining qalinligi, kristall

hosil bo'lishning boshlang'ich va yakuniy haroratlari, shuningdek majburiy sovutish tezligi tartibga solinishi natijasida yuqori sifatli qattiq qorishmalarni olish imkoniyati aniqlandi.

Dastlabki elektron-mikroskopik tahlillar asosida optimal texnologik parametrlar belgilandi: qatlamlarning kristallanish harorati 750°C , epitaksial o'sish tezligi $v_k = 0,15 \text{ mkm/min}$, majburiy sovutish tezligi esa 1 grad/min deb qabul qilindi.

$(\text{Ge}_2)_{1-x-y}(\text{ZnSe})_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$ epitaksial plyonkalarining dastlabki elektrofizik xossalari xona haroratida Van der Pau usuli yordamida o'lchandi. O'tkazuvchanlik turi termozond usulida Hall doimiysi qiymatlari asosida aniqlanib, legirlanmagan qatlamlar asosan kovakli (p-turdagi) o'tkazuvchanlikka ega ekani qayd etildi.

O'lchash natijalariga ko'ra, o'stirilgan plyonkalarining xona haroratidagi solishtirma qarshiligi, zaryad tashuvchilar konsentratsiyasi va harakatchanligi mos ravishda $10 \Omega \cdot \text{sm}$, $1,8 \cdot 10^{15} \text{ sm}^{-3}$ va $368 \text{ sm}^2/(\text{V} \cdot \text{s})$ ni tashkil etdi. Shuningdek, dastlabki elektrofizik parametrlar asosida noasosiy zaryad tashuvchilarning harakatchanligi $4673 \text{ sm}^2/(\text{V} \cdot \text{s})$ deb hisoblandi.

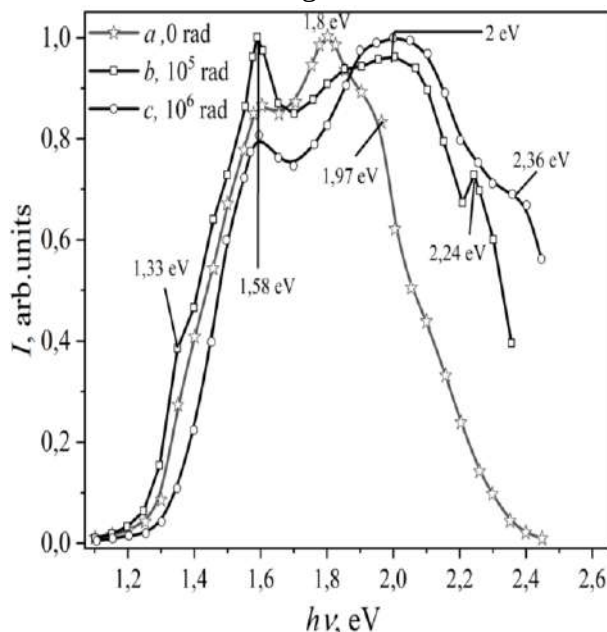
Yarimo'tkazgichli geterotuzilmalarda tok o'tish mexanizmlarini tadqiq etishda, ularga mos omik kontaktlarni tanlash muhim ahamiyat kasb etadi. Agar kontakt sohasi orqali tushayotgan kuchlanish zanjirning boshqa qismlaridagi kuchlanishdan ancha kichik bo'lsa, bunday kontakt amalda omik deb hisoblanadi. Amaliyotda bunday kontaktlarni metall va yarimo'tkazgich oralig'ida kuchli legirlangan yarimo'tkazgich sohasini hosil qilish orqali amalga oshiriladi.

Tadqiqot natijalari.

Geterotuzilmalar asosida ko'ptarkibli o'zgaruvchan energetik sohali quyosh elementlari konsepsiyasini to'ldiradigan ushbu muammoni hal qilishda yangi yondashuv mavjud. Bunday ko'ptarkibli o'zgaruvchan energetik sohali quyosh batareyasining eng istiqbolli modifikatsiyalaridan biri $n\text{-GaAs-}p\text{-(ZnSe)}_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$

geterotuzilmalariga asoslangan. Tadqiqot ishining ushbu bo'limida germaniy nanokristallarining $n\text{-GaAs} - p\text{-(ZnSe)}_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$ geterotuzilmalarining fotoelektrik xossalari gamma nurlanishlarining ta'sirlari o'rganilgan.

$(\text{ZnSe})_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$ epitaksial qatlamlari suyuq fazali epitaksiya usulining majburiy sovutish tizimi yordamida vismut hosil qiluvchi aralashma eritmadan GaAs tagliklariga o'stirilgan. Mazkur geterotuzilma fotosezgirlik spektral bog'lanishi kvars CARLZEISJENA firmasi tomonidan ishlab chiqarilgan kvars oynali monoxromatori bilan jihozlangan optik spektrometr yordamida tekshirildi, bunday qurilma foton energiyasi 1 dan 3 eV gacha bo'lgan diapazonida namunalarni tekshirishga imkon beradi.



1-rasm. $n\text{-GaAs-p-(ZnSe)}_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$ geterotuzilmasining xona xaroratidagi gamma nurlanishlardan keyingi fotosezgirlik spektrlari

1-rasmda (a egri chiziqqa qarang) xona haroratida gamma nurlanishlardan oldingi $n\text{-GaAs-p-(ZnSe)}_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$ geterotuzilmasining fotosezgirlik spektral bog'lanishi keltirilgan. Rasmdan epitaksial plyoknalarining tashkillovchilaridan tor energetik sohali Ge_2 , shuningdek, $\text{GaAs}_{1-\delta}\text{Bi}_\delta$ kvant o'rasining ta'siri tufayli fotovoltaiik effektga faol ta'sirini ko'rsatgani tufayli $n\text{-GaAs-p-(ZnSe)}_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$ geterotuzilmaning fotosezgirlik spektri 1,2 eV

foton energiyasidan boshlab keskin ortishini ko'rish mumkin. Ma'lumki, germaniy ($E_{g,\text{Ge}} = 0.67 \text{ eV}$) eyengetik soha kengligi $\text{GaAs}_{1-\delta}\text{Bi}_\delta$ kvant o' rasi ($E_{g,\text{K.Y.}} = 1.3 \text{ eV}$) hamda ruh selenidi ($E_{g,\text{ZnSe}} = 2.70 \text{ eV}$) larning energetik soha kengliklaridan kichik, ammo fotosezgirlik spektrida germaniyga tegishli sezgirlik cho'qqi yo'qligi, germaniy bilan boyitilgan plyonkaning yupqaligidan ($\sim 5 \text{ mkm}$) dalolat beradi [3].

$n\text{-GaAs-p-(ZnSe)}_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$ geterotuzilmasining fotosezgirlik spektrida 1,58 eV foton energiyali nisbatan sezgarsiz cho'qqi kuzatilgan bo'lib, u $\text{GaAs}_{1-\delta}\text{Bi}_\delta$ kvant o'rasidagi zaryad almashinuvlar va ularning rekombinatsiyalanish xossalari o'zgarishi natijasida namoyon bo'ladi.

Mazkur fotosezgirlik spektridagi eng yuqori cho'qqi 1,8 eV foton energiyasida kuzatiladi (1-rasm, a egri chiziqqa qarang), bu esa o'z navbatida $n\text{-GaAs-p-(ZnSe)}_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$ qattiq qorishmasining man etilgan sohasi kengligi tufayli bo'lishi mumkin. Bu esa o'z navbatida o'stirilgan plyonka tarkibini ko'p miqdorda ruh selenidi atomlari tashkil etayotganligidan dalolat beradi [4]. Bundan tashqari $n\text{-GaAs-p-(ZnSe)}_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$ geterotuzilmaning fotosezgirlik spektrida 1,97 eV foton energiyasiga nisbatan sezgarsiz cho'qqi ham kuzatilgan. Ma'lumki, binar yarimo'tkazgich birikmalarining energetik soha kengligi $\text{ZnSe-}E_{g,\text{ZnSe}} > E_{g,\text{GaAs}_{1-\delta}\text{Bi}_\delta}$ va $E_{g,\text{ZnSe}} > E_{g,\text{Ge}}$ munosabatda bo'ladi. Lekin mazkur qattiq qorishmaning ZnSe, $\text{GaAs}_{1-\delta}\text{Bi}_\delta$ va Ge tashkillovchilari yaqin geometrik o'lcham va zaryad holatlariga ega bo'lgani uchun ko'p miqdordagi ZnSe ga ega bo'lgan $(\text{ZnSe})_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$ qattiq qorishmalarini olish imkonini beradi. Bu esa o'z navbatida bunday turdagi qattiq qorishmalarining man etilgan sohasining torayishiga olib keladi [5]. Shuningdek, 1,97 eV yuqori foton energiyalarida fotosezgirlikning pasayishi kuzatilib, bu p-n o'tishning ajratish to'sig'ining chuqurligi bilan bog'liq bo'lib, bizning hol uchun epitaksial qatlamning qalinligi $\sim 5 \text{ mkm}$ ni tashkil qiladi.

Tadqiq etilayotgan epitaksial plyonkalari uchun zaryad tashuvchilarning diffuziyaviy yugurish yo'li aniqlanganda, u taxminan 4,6 mkm tashkil etdi va bu o'z navbatida ajratish to'sig'ining chuqurligidan kichik ekanligini ko'rsatadi. Mazkur holat fotoduktsiyalangan nomuvozanat zaryad tashuvchilarning rekombinatsiyasini kuchaytiradi. Bundan tashqari, tadqiq etilayotgan qattiq qorishmlarni vismut hosil qiluvchi aralashma eritmadan o'stirilgani tufayli epitaksial plyonkalar kristall panjarasiga kam miqdorla vismut atomlari kirib qolib, sayoz donor sathlar hosil qilishi ham mumkin [6].

1-rasmda (b egri chiziqqa qarang) xona haroratida 10^5 rad gamma nurlanishlardan keyingi $n\text{-GaAs-p-(ZnSe)}_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$ geterotuzilmasining fotosezgirlik spektrida bir qator sezilarli o'zgarishlar kuzatilishi taqdim etilgan. Rasmdan mazkur tuzilmalarning fotosezgirlik spektrlarining qisqa va uzun to'lqinlar tomon siljiganini, ya'ni kengayganini ko'rish mumkin. Bu esa o'z navbatida gamma nurlanish ta'sirida geterotuzilmalarning hajmiy va sirtiy sohalarida yangi chuqur sathli rekombinatsiya markazlari hosil bo'lishidan dalolat beradi.

Shuningdek, yuqorida ta'kidlanilganidek, $\text{GaAs}_{1-\delta}\text{Bi}_\delta$ kvant o'rasining fotovoltaiik effektga faol ta'sirini ko'rsatgani tufayli $n\text{-GaAs-p-(ZnSe)}_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$ geterotuzilmaning fotosezgirlik spektri 1,2 eV foton energiyasidan boshlab keskin ortishini ko'rish mumkin. Bundan tashqari 10^5 rad gamma nurlanishdan so'ng 1,3 eV foto energiyali $\text{GaAs}_{1-\delta}\text{Bi}_\delta$ kvant o'rasiga tegishli fotosezgirlik cho'qqining sezilarli o'zgarishi kuzatilgan. Bu esa o'z navbatida gamma nurlanishdan keyin bo'linish chegaralarida turli nuqsonlar orqali o'tayotgan fototok tashuvchilarini kvant o'raga samaraliroq o'tayotganidan, tuzilmaning sirtiy sohalarida yutishni ortganidan dalolat beradi [7]. Shuningdek, GaAs birikmasini As atomlari bilan Bi atomlarining oz miqdorda almashinishi tufayli GaAs valent sohasida rezonans holatlar keltirib chiqaradi va ta'qiqlangan soha kengligini (1,3 eV < 1,42 eV) kamaytirdi. 10^5 rad gamma nurlanishdan keyin Bi atomlarining kichik o'lchamli klasterlanishi GaAs birikma

kristal panjarasida kichik o'lchamdagi zo'riqishlarning ortishi (energetik sathlarni qayta tashkillanishi) tufayli tadqiq etilayotgan tuzilmaning $\text{GaAs}_{1-\delta}\text{Bi}_\delta$ kvant o'rasiga tegishli mahalliyashgan holatlar miqdori ortadi, natijada fotozergilik spektridagi 1,3 eV foto energiya aniqroq kuzatilishiga olib keladi.

10^5 gamma nurlanishdan keyingi tuzilmalarning fotosezgirlik spektridagi eng yuqori cho'qqi 1,58 eV foton energiyasida kuzatilib, nurlantirilmagan tuzilmalarniikiga (1,8 eV) nisbatan uzun to'lqilar tomon siljigan (1-rasm, b egri chiziqqa qarang). Yuqorida ta'kidlab o'tilganidek, gamma nurlanishdan keyin $\text{GaAs}_{1-\delta}\text{Bi}_\delta$ kvant o'ralar orqali energiya almashinuv samarasi ustunlashadi. Bu esa o'z navbatida foton energiyasi kamroq bo'lgan fototok o'tishlarini rag'batlantiradi [8]. Bundan tashqari $\text{GaAs}_{1-\delta}\text{Bi}_\delta$ kvant o'rasiga tegishli mahalliyashgan holatlar miqdori ortishi ham tuzilmalarning fotosezgirlik cho'qqisi 1,8 eV dan 1,58 eV ga tomon siljishiga sababchi bo'ladi [8].

10^5 rad gamma nurlanishlardan keyingi $n\text{-GaAs-p-(ZnSe)}_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$ geterotuzilmasining fotosezgirlik spektrida 2 eV foton energiyali cho'qqi kuzatilib, u nurlantirilmagan tuzilmalardagiga (1,97 eV) nisbatan qisqa to'lqinlar tomon siljigan holda namoyon bo'lgan. Bu esa o'z navbatida nurlanish ta'sirida elektron zichligi ortishi, kristall pajarada qo'shimcha shakllanadigan nuqsonlarning nisbatan tartiblanishi hamda o'tish qatlamining maxalliy deformatsiyasi natijasida yuzaga kelishidan dalolat beradi. Shuning uchun mazkur xolat yuqori solishtirma qarshilikka ega bo'lgan o'tish qatlamidagi energetik holatlarning qayta taqsimlanishi, shuningdek, Kvant-Konch-Shtark va Moss–Byurshteyn effektlarining birgalikdagi ta'siri bilan izohlanadi. Bundan tashqari 10^5 rad gamma nurlanishlardan keyingi tadqiq etilayotgan tuzilmalarning fotosezgirlik spektri 2,24 eV foton energiyali cho'qqi ham kuzatilgan. Mazkur cho'qqi asosan ZnSe qatlami va geteroo'tishnig chegara sohalarida turli nuqsonlar bilan bog'liq yuqori energiyali elektron–kovak o'tishlar natijasida kuzatilayotgan bo'lishi mumkin. Bunday xolat 10^5 rad gamma nurlanishlar ta'sirida yuqori solishtirma qarshilikka ega bo'lgan o'tish qatlamidagi yutish sohalarining faollashishi, mahalliy (subkristallitlarning bo'linish

chegaralarida ZnSe–GaAsBi qayta shakllangan) nuqsonlar ortishi bilan izohlanadi [9].

1-rasmda (s egri chiziqqa qarang) xona haroratida 10^6 rad gamma nurlanishlardan keyingi $n\text{-GaAs-p-(ZnSe)}_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$ geterotuzilmasining fotosezgirlik spektri hamda uning eng yuqori choʻqqisi 2 eV foton energiyasida kuzatilib, nurlantirilmagan tuzilmalarniikiga (1,8 eV) nisbatan qisqa toʻlqilar tomon siljishi (1-rasm, s egri chiziqqa qarang) kuzatilgan. Bu esa oʻz navbatida gamma nurlanishlar taʼsirida tuzilma hajmida yuzaga kelgan elektr maydonlarning erkin zaryad tashuvchilar tomonidan kompensatsiyalanishi (ekranlashishi) hamda kristall panjaradagi buzilishlarni kamayishi natijasida yuqori energiyali fotonlarga sezgirlik ortishidan dalolat beradi [10].

Bundan tashqari 10^6 rad gamma nurlanishlar taʼsir ettirilgan tuzilmalarda 2,36 eV foton energiyali choʻqqi ham namoyon boʻlgan. [11] tadqiqot ishi muallifining taʼkidlashicha, suyuqfazali epitaksiya usuli yordamida olingan $(\text{GaAs})_{1-x}(\text{ZnSe})_x$ qattiq qorishmlarning fotoluminessensiya spektri 400 nm dan 760 nm boʻlgan deyarli butun koʻzga koʻrinuvchi sohani qamrab olagan. [11] tadqiqot ishi mualliflari esa GaSe birikmasi tufayli chuqur energetik sathlar ($h\nu_{\max} = 1.88$ eV), ZnAs ($h\nu_{\max} = 2.15$ eV) i ZnSe ($h\nu_{\max} = 2.69$ eV) paydo boʻlishini aniqlaganlar. Bizning hol uchun 1.92, 2.3 va 2.63 eV foton energiyalarida uchta fotosezgirlik choʻqqilari kuzatilib, GaSe va ZnAs birikmalarining qattiq qorishma kristall panjarasida turli modifikatsiyalari tufayli birinchi va ikkinchi fotosezgirlik choʻqqilari qisman qisqa toʻlqinlar tomon ogʻgan.

Xulosa

Shunday qilib, $n\text{-GaAs-p-(ZnSe)}_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$ geterotuzilmalarning fotosezgirlik spektrlari gamma nurlanishlar taʼsirida qisqa toʻlqinlar tomon siljishi, turli nuqsonlarning taqiqlangan sohani kengaytirishi hamda qoʻshimcha mahalliydashgan energetik sathlarning hosil boʻlishi bilan izohlanadi. Tuzilmalar gamma nurlari bilan nurlantirilganda yangi rekombinatsiya markazlari paydo boʻlishi natijasida fotosezgirlik spektrida yuqori foton

energiyasiga ega yangi choʻqqining paydo boʻlishi aniqlandi.

Tadqiqot natijalari shuni koʻrsatadiki, gamma nurlanishlar geterotuzilmalarning oʻtish qatlamida elektr maydonlarning qayta shakllanishiga, zaryad tashuvchilar konsentratsiyasining ortishiga va foton yutilish sohalarining faollashishiga sabab boʻladi. Ushbu jarayonlar Kvant-Konch–Shtark va Moss–Byurshteyn effektlarining birgalikdagi taʼsiri natijasida kechadi. Natijada $\text{GaAs}_{1-\delta}\text{Bi}_\delta$ kvant oʻralaridagi Bi atomlarining klasterlanishi rezonans holatlarning barqarorlashuviga olib keladi va fotosezgirlik choʻqqilarining energiya qiymatlari 1,8 eV dan 1,58 eV gacha siljishi kuzatildi.

Olingan natijalar asosida gamma nurlanish taʼsirida $n\text{-GaAs-p-(ZnSe)}_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$ geterotuzilmalarning energetik sohalarini boshqarish va fotosensor qurilmalarning ishlash samaradorligini oshirish imkoniyati aniqlandi. Umuman olganda, gamma nurlanishlar taʼsirida hosil boʻladigan strukturaviy va energetik oʻzgarishlar ushbu geterotuzilmalarning fotoelektrik xususiyatlarini yaxshilab, yuqori samarali optoelektron va fotodetektor tizimlarini yaratish uchun keng imkoniyatlar ochadi.

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INFLUENCE OF A STRONG MAGNETIC FIELD ON FERMI ENERGY OSCILLATIONS IN TWO-DIMENSIONAL SEMICONDUCTOR MATERIALS

G. Gulyamov¹, U.I. Erkaboev², R.G. Rakhimov^{1,2}, N.A. Sayidov², J.I. Mirzaev²

¹ Namangan Engineering-Construction Institute, 160103, Namangan, Uzbekistan

² Namangan Institute of Engineering and Technology, 160115, Namangan, Uzbekistan

Abstract: This article discusses the effect of a quantizing magnetic field and temperature on Fermi energy oscillations in nanoscale semiconductor materials. A generalized mathematical expression is obtained for calculating the dependence of the Fermi energy oscillations on the magnetic field, temperature, and thickness of the quantum well. It is shown that the Fermi energy in nanoscale semiconductor materials under a quantizing magnetic field is quantized. The proposed theory explains the experimental results in two-dimensional semiconductor structures with a parabolic dispersion law.

Keywords: quantizing magnetic field, temperature, Fermi energy, nanoscale semiconductors, quantum well.

Аннотация: В данной статье рассмотрено влияние квантующего магнитного поля и температуры на осцилляции энергии Ферми в наноразмерных полупроводниковых материалах. Получено обобщенное математическое выражение для вычисления зависимости осцилляции энергии Ферми от магнитного поля, температуры и толщины квантовой ямы. Показано, что энергия Ферми в наноразмерных полупроводниковых материалах под влиянием сильного магнитного поля квантована. Предложенная теория объясняет экспериментальные результаты в двумерных полупроводниковых структурах с параболическим законом дисперсии.

Ключевые слова: квантующее магнитное поле, температура, энергия Ферми, наноразмерные полупроводники, квантовая яма.

Аннотация: Ушбу мақолада нано ўлчамли яримўтказгичли материалларда Ферми энергияси осцилляцияларига квантловчи магнит майдон ва ҳароратнинг таъсири кўрилган. Ферми энергияси осцилляцияларининг магнит майдон, ҳарорат ва квант ўранинг қалинлигига боғлиқлигини ҳисоблаш учун умумлашган математик ифода олинган. Кучли магнит майдон таъсирида наноўлчамли яримўтказгичли материалларда Ферми энергиясининг квантланиши кўрсатилган. Параболик дисперсия қонуни учун икки ўлчовли яримўтказгичли структураларда тавсия этилган назария билан эксперимент натижалари тушунтирилган.

Калит сўзлар: квантловчи магнит майдон, ҳарорат, Ферми энергияси, наноўлчамли яримўтказгичлар, квант ўра.

INTRODUCTION

At present, interest in studying the properties of two-dimensional electronic systems is due to the prospects for their application in nanoscale semiconductor structures. In such systems, the quantum dimensional quantities of the dependence of the characteristics have, as a rule, an oscillating character [1-9]. In two-dimensional semiconductors, macroscopic energy characteristics such as the density of states, effective masses of electrons, and the Fermi energy depend on the thickness of the quantum well. It is assumed that the size of the thickness of the materials d will be commensurately equal

to the de Broglie wavelength of the electron in low-dimensional semiconductors. Let us consider a degenerate (for definiteness - electron) Fermi gas in a magnetic field that is weak compared to the Fermi energy. In the absence of a field, each state is occupied by two electrons that differ in the projection of the spin to the chosen direction, the energy is determined only by the kinetic energy (momentum) and does not depend on the projection of the spin. If a magnetic field is applied to this system, then one of the states (with a magnetic moment directed along the field) becomes energetically more favorable and there should be more electrons in this state.

As already noted, since the motion in cyclotron orbits is finite, in accordance with the general rules of quantum physics, it must be quantized. Qualitatively, the quantization result can

be obtained using the Bohr-Sommerfeld rule. When moving in a cyclotron orbit $p \cdot 2\pi r = 2\pi m\omega_c r^2 = n\hbar$. Where is the radius of the quantized orbit $r_n = \sqrt{\frac{n\hbar}{2\pi m\omega_c}} = \sqrt{n \frac{\hbar c}{eB}}$, and the

energy of the quantized motion $E_n = \frac{p_n^2}{2m} = \frac{n^2 \hbar^2}{2mr_n^2} = n\hbar \frac{eH}{2mc} = n \frac{\hbar\omega_c}{2}$. This answer gives an idea of the

effect in order of magnitude, but, as we will see below, it differs from the exact one by a factor of two. The exact solution to this problem requires the solution of the Schrödinger equation in a magnetic field. Let us consider electrons as non-interacting, then we get a simple one-particle problem. We will use the fact that in a magnetic field the momentum of a particle is renormalized

$\vec{p} \rightarrow \vec{p} - \frac{e}{c} \vec{A}$. Where, $\vec{A}$ is the vector potential of the magnetic field ($\vec{B} = \text{rot } \vec{A}$), which for this

problem is conveniently chosen in the form $\vec{A} = (0, Bx, 0)$ (the field is directed along Z). The Schrödinger equation for a free electron has the form:

$$\frac{1}{2m} (p_x^2 + (p_y - \frac{e}{c} Bx)^2 + p_z^2) \psi(x, y, z) = E \psi(x, y, z) \quad (1)$$

solution in the form $\psi(x, y, z) = e^{ik_y y} e^{ik_z z} \psi(x)$. The variable z immediately separates and gives

just free movement along the Z axis with a contribution to the energy $\frac{\hbar^2 k_z^2}{2m}$. For motion in the plane, by substitution, we obtain

$$\frac{1}{2m} (-\hbar^2 \psi'' + (\hbar^2 k_y^2 - 2k_y \hbar \frac{e}{c} Bx + \frac{e^2}{c^2} B^2 x^2) \psi) = \omega \psi \quad (2)$$

$$-\frac{\hbar^2}{2m} \psi'' + \frac{e^2 B^2}{2mc^2} (x - \frac{\hbar c k_y}{eB})^2 \psi = \omega \psi \quad (3)$$

The equation coincides with the equation of a harmonic oscillator with an equilibrium position at the point $x_0 = \frac{\hbar c k_y}{eB}$, called the position of the leading center. The “hardness” of the

oscillator is $\kappa = \frac{e^2 B^2}{mc^2}$, respectively, the characteristic frequency that determines the quantum of

the oscillator energy $\omega_c = \sqrt{\frac{\kappa}{m}} = \frac{eB}{mc} = \frac{m_0}{mc} \frac{2\mu_B B}{\hbar}$, which coincides with the classical cyclotron

frequency (m_0 is the free electron mass entering the Bohr magneton). The energy of the n-th level

is $W_n = \hbar\omega_c(n + \frac{1}{2})$, total energy $E(n, k_z) = \hbar\omega_c(n + \frac{1}{2}) + \frac{\hbar^2 k_z^2}{2m}$. Let's introduce the magnetic length

$l_B = \sqrt{\frac{\hbar c}{eB}}$, taking this designation into account $x_0 = l_B^2 k_y$. The magnetic length sets the characteristic

scale by which the electron moves away from the leading center; it can be interpreted in the semiclassical approach as a value of the order of the radius of the cyclotron orbit for the first Landau level. In a field of 10 T, the magnetic length is $0.81 \cdot 10^{-6}$ cm. The magnetic flux penetrating

the cyclotron orbit, $F_0 = \pi l_B^2 B = \frac{\pi \hbar c}{e} = 2.05 \cdot 10^{-7} \text{ KOe} \cdot \text{sm}^2$ called the flux quantum. The energy

levels found are the Landau levels of electrons in an external magnetic field. Note that the energy of levels with different coordinates of the leading center coincides, the Y-component of the wave vector enters only in the coordinate of the leading center.

Thus, to find the equilibrium configuration, we need to minimize the sum of the kinetic energy of electrons of two polarizations and the Zeeman energy, taking into account the constancy

of the total number of electrons. If n_0 is the total concentration of electrons, $n_{\pm} = \frac{n_0}{2} \pm \delta n$ – concentration of favorable and unfavorable polarizations (this designation automatically preserves the total number of electrons), then the radii of Fermi spheres for electrons of favorable and unfavorable polarizations $k_{\pm} = \sqrt[3]{6\pi^2(\frac{n_0}{2} \pm \delta n)}$ (note that under the root $6\pi^2$, not $3\pi^2$, since we now consider the Fermi radii without spin degeneracy). Recalling that the average kinetic energy of a degenerate Fermi gas is $\frac{3}{5}E_F$, we can write down the total energy, which is then minimized with respect to δn :

$$E = n_+ \frac{3}{5} \frac{\hbar^2}{2m} k_+^2 + n_- \frac{3}{5} \frac{\hbar^2}{2m} k_-^2 - 2\delta n \mu_B B \quad (4)$$

The g-factor is assumed to be 2. The calculations can be significantly simplified using the concept of the density of states. Indeed, if $\mu_B \ll E_F$ (which is true for metals in all reasonable fields), then the redistribution of electrons will occur only near the Fermi surface. Density of states (the number of states in a unit energy range) at the Fermi surface for a degenerate Fermi gas.

$D(E_F) = \frac{3n}{2E_F}$, where n is the total concentration of electrons. This density of states is total, taking into account both polarizations; for each of the polarizations, the number of states is half. When the field is turned on, the electron energy will change by $\pm \mu_B B$, respectively, and the electron concentrations of the corresponding polarizations changed by $\pm \frac{D}{2} \mu_B B$. From where we immediately find the total magnetic moment of a unit volume $M = D \mu_B^2 B$ and, finally, the susceptibility of a unit of volume $\chi_{para} = D \mu_B^2 = \frac{3n \mu_B^2}{2E_F} = \frac{m \mu_B^2}{\hbar^2} \sqrt[3]{\frac{3n}{n^4}}$. The resulting magnetic moment

is paramagnetic (directed along the applied field), independent of temperature (at $k_T \ll E_F$). This effect is called Pauli paramagnetism. Note that the mass entering into the paramagnetic susceptibility is the effective mass of the electron (it is the effective mass that enters into the density of states on the Fermi surface). Landau levels describe quantized motion in a plane perpendicular to the field. For physical applications, it is essential to check how the density of states will change due to the appearance of this quantization. To find the conditions for quantization, consider a sample in the form a rectangular parallelepiped with edges L_x , L_y and L_z , all lengths are considered to be much greater than the magnetic length. The coordinates of the leading center depend on k_y . At the same time, the center itself must lie in the range from 0 to L_x ; otherwise, due to the rapid decrease in the wave function of the oscillator, the probability of detecting such a state in the sample will tend to zero. By imposing periodic boundary conditions along Y and Z , we obtain the usual condition that one state has an element of the phase volume of the k -space $\frac{2\pi}{L_{y,z}}$. Therefore, in total in the XY plane for each value of k_z there is $\frac{1}{l_B^2} \frac{L_x L_y}{2\pi}$ states at

each Landau level. And in the interval dk_z at each Landau level there will be $dN = \frac{V}{(2\pi)^2} \frac{1}{l_B^2} dk_z$ states.

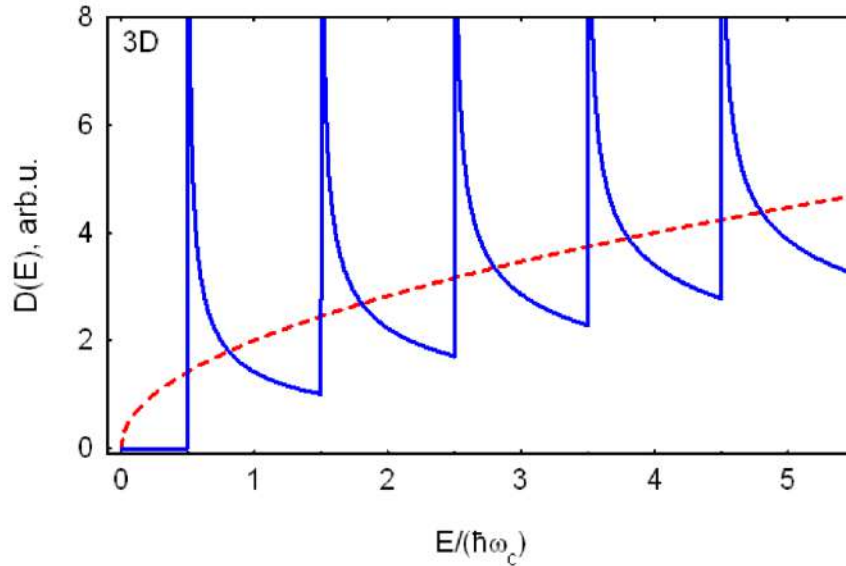


Fig. 1. Schematic representation of the density of states in a three-dimensional Fermi gas with allowance for the quantization of Landau levels. The dotted line is the density of states in the absence of a field.

To find the density of states as a function of energy, consider the n th Landau level, the electron energy at this level $E(n, k_z) = \hbar\omega_c(n + \frac{1}{2}) + \frac{\hbar^2 k_z^2}{2m}$ has a minimum value $\hbar\omega_c(n + \frac{1}{2})$, each value is twofold degenerate in the k_z direction and twofold degenerate in spin (we neglect the interaction of the spin moment with the field, considering the purely orbital motion of the electron). Whence the contribution of this level to the density of states:

$$D_n(\varepsilon) = 4 \frac{V}{(2\pi)^2} \frac{1}{l_B^2} \frac{d|k_z|}{d\varepsilon} = 4 \frac{V}{(2\pi)^2} \frac{1}{l_B^2} \frac{\sqrt{2m}}{\hbar} \frac{1}{\sqrt[3]{\varepsilon - \hbar\omega_c(n + \frac{1}{2})}} \quad (5)$$

total density of states $D(\varepsilon) = \sum_{n=0}^{\infty} D_n(\varepsilon)$. (Fig.1)

Thus, instead of a monotonic dependence of the density of states on energy, periodic sharp maxima appear in it. As we recall, many of the properties of a metal depend on the density of states at the Fermi level - therefore, we can expect that these properties will change dramatically depending on whether the position of the Fermi level falls on the peak of the density of states or on one of the minima.

The presence of a spin in the electron will lead to the splitting of the Landau level into two spin sublevels. If the motion in the cyclotron orbit is independent of the spin degree of freedom, then this will simply lead to a shift of the spin sublevels by $\pm \frac{g}{2} \mu_B B$ the position of the unsplit level, where g is the electronic g -factor. For free electrons $g = 2$ and $m = m_0$, which would lead to the fact that the Zeeman splitting $\Delta E_{Zeeman} = g\mu_B B$ would coincide with the cyclotron $\Delta E_c = \hbar\omega_c = 2\mu_B B$. As a result, the energies of the sublevel of the n th Landau level with the Zeeman addition $+\mu_B B$ and the sublevel of the $(n + 1)$ th Landau level with the Zeeman addition $(-\mu_B B)$ would coincide. It is necessary, however, to bear in mind that we are not talking about a free electron, but about a quasiparticle in a solid. For such quasiparticles, the cyclotron splitting is determined by the effective mass $\hbar\omega_c = \frac{m_0}{m} 2\mu_B B$, a g -factor can also change quite strongly

(in problems of assignment there are examples with values of the g -factor of an electron from ≈ 1 to ≈ 60). Therefore, the relationship between cyclotron and Zeeman splitting can vary within fairly arbitrary limits, and a general approach to constructing the structure of electron levels in a

magnetic field with allowance for spin sublevels is impossible. In the literature, the term “Landau level” can be understood as a problem without taking into account spin splitting, which we considered, and a spin sublevel with a certain value of S_z .

Let us note on a qualitative level that during the orbital motion of electrons in a cyclotron orbit, each electron creates a magnetic field directed against the external one. That is, the orbital motion along cyclotron orbits should lead to a diamagnetic response of the system of electrons - that is, the effect opposite in sign to the Pauli paramagnetism considered above. To find the effect, you need to calculate the total energy. Consider the case of low temperature and weak field $\mu_B B \ll T \ll \varepsilon_F$. The smallness of the temperature makes it possible to replace the Fermi distribution with a step, the smallness of the temperature and field makes it possible to neglect the change in the Fermi energy, the smallness of the field in comparison with temperature “smears out” the irregularities in the dependence of the density of states on energy by thermal fluctuations and allows the density of states to be considered a smooth function. We neglect the spin splitting.

For compactness, let us introduce the notation $\varepsilon_n = \hbar \omega_c (n + \frac{1}{2})$. Then the energy

$$E = \int_0^{\varepsilon_F} (\varepsilon - \varepsilon_F) D(\varepsilon) d\varepsilon = \int_0^{\varepsilon_F} \sum_n (\varepsilon - \varepsilon_F) D_n(\varepsilon) d\varepsilon \quad (6)$$

For the convenience of subsequent calculations, the Fermi energy has been subtracted, which is not important since we are interested in the field-dependent correction. Taking into account the admissible values, the integration of each term must be carried out on ε_n and the summation is limited from above by the condition $\varepsilon_n < \varepsilon_F$. We integrate piece by piece:

$$f(n) = \int_{\varepsilon_n}^{\varepsilon_F} (\varepsilon - \varepsilon_F) D_n(\varepsilon) d\varepsilon = \frac{V}{\pi^2 l_B^2} \frac{\sqrt{2m}}{\hbar} \int_{\varepsilon_n}^{\varepsilon_F} (\varepsilon - \varepsilon_F) d\sqrt{\varepsilon - \varepsilon_F} = -\frac{V}{3\pi^2 l_B^2} \frac{\sqrt{2m}}{\hbar} (\varepsilon_F - \varepsilon_n)^{\frac{3}{2}} \quad (7)$$

In a small field, the maximum value of n is large. For large n , summation can be replaced by integration using a variation of the Euler-Maclaurin formula:

$$\sum_{n=0}^{n_{\max}} f(n) = \int_{-\frac{1}{2}}^{n_{\max} + \frac{1}{2}} f(n) dn - \frac{1}{24} \left[f'(n_{\max} + \frac{1}{2}) - f'(-\frac{1}{2}) \right] \quad (8)$$

Further, we note that during integration it will be necessary to change the variable from n to ε_n and, as a result, the integral will not depend on the field - hence, it will give (all or partly) the total energy of the Fermi gas in the absence of a field, which is not of interest to us. Derivative

$$\frac{df}{dn} = \frac{df}{d\varepsilon_n} \cdot \frac{d\varepsilon_n}{dn} = \frac{2V}{3\pi^2 l_B^2} \frac{\sqrt{2m}}{\hbar} \frac{3}{2} \sqrt{\varepsilon_F - \varepsilon_n} \hbar \omega_c \quad (9)$$

$$\varepsilon_{-1/2} = 0, \quad \varepsilon_{n_{\max} + 1/2} \approx \varepsilon_F$$

Thus, for the field-dependent part of the energy:

$$E = E_0 + \frac{1}{24} \frac{V}{\pi^2} \frac{eB}{\hbar c} k_F \hbar \frac{eB}{mc} = E_0 + \frac{V}{24\pi^2} \frac{e^2}{mc^2} \sqrt[3]{3\pi^2 n} \cdot B^2 = E_0 + \frac{V}{6} \frac{m\mu_B^2}{\hbar^2} \sqrt[3]{\frac{3n}{\pi^4}} \cdot B^2 \quad (10)$$

We see that the energy really grows with the field, that is, there is a diamagnetic contribution. This contribution is called Landau diamagnetism. By double differentiation with respect to the field, we find the diamagnetic susceptibility of a unit volume:

$$\chi_{dia} = -\frac{1}{3} \frac{m\mu_B^2}{\hbar} \sqrt[3]{\frac{3n}{\pi^4}} = -\frac{1}{3} \chi_{para} \quad (11)$$

Thus, the diamagnetic contribution is three times less than the paramagnetic one. It would seem from our conclusion that every electron gas must be paramagnetic. However, it is known from experiment that some metals (copper, gold) are diamagnetic. One of the reasons for the diamagnetism of a metal is that, generally speaking, for a metal with a complex Fermi surface and

a non-quadratic spectrum, the effective masses in calculating the paramagnetic susceptibility ("dynamic" effective mass at the Fermi surface) and diamagnetic susceptibility (cyclotron mass) can differ markedly both in one and the other direction.

As is known, the energy spectrum of electrons has highly variable properties depending on the relative position of the Fermi level with respect to the Landau levels in two-dimensional semiconductors in the presence of a quantizing magnetic field. All electron gases have a single Fermi level μ , which at absolute zero temperature determines the level of filling the energy bands with electrons. As is known from the experimental and theoretical data [10-12], in two-dimensional semiconductors, the Fermi surface at absolute temperature is characterized by rather high amplitudes of the Fermi energy oscillations (μ). But, for a three-dimensional electron gas, the μ oscillations will be very weak, even at low temperatures. In three-dimensional semiconductors, μ changes only linearly, as in classical magnetic fields.

When studying the electronic and magnetic properties of two-dimensional electronic systems, an important characteristic is the Fermi energy, which determines the main contribution to micro and nanoscale semiconductors. Therefore, the purpose of this work is to analyze the effect of a quantizing magnetic field on dimensional oscillations of the Fermi energy in two-dimensional semiconductor structures.

METHODS

For low-dimensional semiconductor structures and metals, the condition $\varepsilon_F \gg \hbar\omega_c$ is always satisfied, which means that a large number of Landau levels are always filled for metals. When the field changes, the distance between these levels' changes and when the condition $\varepsilon_F = \hbar\omega_c(n + \frac{1}{2})$. The next Landau level crosses the Fermi surface. Due to the large number of occupied levels, the situation when there is $n \gg 1$ or $(n-1) \gg 1$ levels under the Fermi surface are almost the same - only the location of the Fermi surface relative to the nearest Landau level is important. Consequently, we can expect the appearance of some oscillations in the properties of the metal when the next Landau level passes through the Fermi surface.

A necessary condition for observing such oscillations is the smallness of the temperature in comparison with the cyclotron splitting of levels - for observing oscillatory effects, $T \ll \hbar\omega_c$ is required. Before the development of modern techniques such as APRES, oscillation effects, along with cyclotron resonance, were the main methods for studying Fermi surfaces [1]. Now they are used to study the properties of exotic conductors [8] due to the technical simplicity of their implementation and the possibility of placing the sample in conditions of ultra-low temperatures, strong magnetic fields [9], [10], [11], [12] or high pressures [13].

Conductivity oscillations associated with the passage of Landau levels through the Fermi surface are called the Shubnikov-de Haas effect. This phenomenon was discovered in 1930 on bismuth by the Soviet physicist L.V. Shubnikov and the Dutch physicist W. de Haas working together in Leiden. The physical reason is obvious: the conductivity of the metal $\sigma = \frac{1}{3}e^2(v^2\tau D(\varepsilon))_\mu$ is determined, among other things, by the density of states at the Fermi surface. Therefore, a periodic change in the density of states (Fig.1) when the Landau level crosses the Fermi surface will lead to resistance oscillations. Conductivity maxima are observed provided that the Fermi energy coincides with the next extremum on the $D(\varepsilon)$ dependence (Fig.2).

$$\varepsilon_F = \hbar\omega_c(n + \frac{1}{2}) = \hbar \frac{eB}{mc}(n + \frac{1}{2}) \quad (12)$$

That is, the experimentally measured dependence $\sigma(B)$ should have regular equidistant maxima when plotted as a function of the inverse field $1/B$. Oscillations in the coordinates of the reverse field are traditionally characterized by the "frequency" $F = \frac{\varepsilon_F mc}{\hbar e}$, measured in units of magnetic field strength (tesla). In the presence of several cyclotron masses, several "frequencies"

and “beats” will be observed on the σ ($1/B$) plot. An example of such data measured on a compound from the “heavy fermion” class. The measurements were carried out in fields up to 20 T at temperatures up to 35 mK. It can be seen that, against the background of a general (monotonic up to almost maximum field) growth of the resistance with the field, resistance oscillations with an amplitude of about 1% arise. The increase in the amplitude of the oscillations with the field is due to the fact that the distance between the Landau levels increases with increasing field, and thermal fluctuations have an ever-smaller effect. For the same reason, the amplitude of the oscillations increases with decreasing temperature. Thus, an analysis of the temperature dependence of the oscillation amplitude (for this purpose, the Lifshitz-Kosevich theory was developed) makes it possible to extract information on the magnitude of the splitting of the Landau levels, that is, on the cyclotron mass. The observed oscillations correspond to an effective electron mass of about 13 masses of a free electron. An example of the observation of Shubnikov oscillations in fields up to 60 T (pulsed magnetic field at the National Laboratory for High Magnetic Fields in Talahasi, Florida, USA) and at high pressures of more than 10,000 atmospheres, observed in related quasi-two-dimensional organometallic conductors.

When deriving the diamagnetic response of the Fermi gas, we assumed the temperature to be large in comparison with the field, which made it possible to consider the density of states to be smeared by thermal fluctuations to smoothness and to use an approximate formula for summation when calculating the energy. Under conditions when the cyclotron splitting exceeds the temperature, this cannot be done, the exact calculations are rather cumbersome and we will not present them [3]. We only point out that, within the framework of our calculations, small energy oscillations will arise when the difference in the position of the last Landau level from the Fermi energy is taken into account: in the previous calculation, we set $\varepsilon_{n_{\max}} + 1/2 \approx \varepsilon_F$, which completely vanished one of the terms. Small oscillations of energy after differentiation can turn into noticeable oscillations of magnetization. The periodicity of these oscillations is determined in the same way as for the resistance oscillations discussed above. For the experimenter, this method is of interest due to the even greater simplicity of measurement than resistance - when measuring under extreme conditions, it is not necessary to lead a wire to the sample. When placed in a magnetic field gradient, the bending of the cantilever is proportional to the magnetization of the sample. In the experiment, field modulation was used to increase the sensitivity; therefore, the observed signal is proportional to the derivative of the magnetic susceptibility. The Fermi surface of gold consists of spheres connected by constrictions at the boundaries of the first Brillouin zone. Therefore, in different orientations of the magnetic field, different extreme cross sections of the Fermi surface contribute to the cyclotron mass. In one of the cases, there is an overlap of two oscillation frequencies. Without going into details of the analysis, we note that this method makes it possible to detect the presence of specific features of the Fermi surface and the presence of carriers with different effective masses, and to determine the effective (cyclotron) mass of carriers and their concentration.

It is known that in k -space the isoenergy surfaces $E(k)=const$ are closed and are represented in the form of a sphere. The allowed energy states have a constant density $V/8\pi^3$ and are distributed in k -space. Here, V is the volume of the crystal. Since two opposite orientations of the spin of the electron state are responsible for each value of k , the wave numbers of all states that will be filled have values no more than k_F in the volume of the crystal V , according to the Pauli principle, and k_F is determined [13]:

$$\frac{4}{3}\pi k_F^3 \frac{2V}{8\pi^3} = N^{3d} \quad (13)$$

$$\text{From here} \quad k_F = \left(\frac{3\pi^2 N^{3d}}{V} \right)^{1/3} \quad (14)$$

Here, N^{3d} is the number of electrons for a three-dimensional electron gas.

If the system of electrons is due to the Fermi-Dirac statistics, then the energy in the ground state, i.e. at absolute temperature, called maximum:

$$E_F = \frac{\hbar^2 k_F^2}{2m} \quad (15)$$

E_F - Called the Fermi energy for 3D electron gas. The Fermi surface will have a spherical shape with a radius of k_F for the isotropic dispersion law. The expressions given above are obtained only for bulk materials and do not consider changes in the Fermi energy oscillations in two-dimensional electron gases.

Now, consider the dependence of the Fermi energy on the quantizing magnetic field in two-dimensional electron gases. In the absence of a magnetic field in two-dimensional electron gases, the electron energy is quantized along the Z-axis, so the electron moves freely only in the XY plane. These quantizations are called dimensional quantization. But, if the magnetic induction B is directed perpendicular to the XY plane, then the free energy of the electron is also quantized along the XY plane.

The question arises: how will the Fermi energy change in two-dimensional electron gases in the presence of a quantizing magnetic field.

For a 2D electron gas, the allowed energy states have a constant density $S/4\pi^2$ and are distributed in the XY plane. Here, S is the surface area crystal. Then, using formulas (1) and (2), we determine the electron concentration for a two-dimensional electron gas:

$$N^{2d} = 4\pi k_F^2 \frac{2L^2}{4\pi^2} \quad (16)$$

$$\text{Hence: } k_F^{2d} = \frac{1}{L} \left(\frac{\pi N^{2d}}{2} \right)^{\frac{1}{2}} \quad (17)$$

Now, we calculate the Fermi energies for a two-dimensional electron gas with a parabolic dispersion law. Substituting (5) to (6), one can determine the Fermi energies in two-dimensional electron gases in the absence of a magnetic field:

$$\mu^{2d} = \frac{p_\mu^2}{2m} = \frac{\pi \hbar^2 N^{2d}}{4mL^2} \quad (18)$$

Here, N^{2d} is the concentration of electrons for a two-dimensional electron gas, L^2 is the surface of the plane of motion, p_μ is the Fermi momentum.

In the motion of a plane perpendicular to the magnetic field, the classical trajectories of electrons are circles. In quantum physics, such trajectories of electrons (periodic rotation of an electron) are equidistant discrete Landau levels:

$$E_n = \hbar \omega_c \left(n_L + \frac{1}{2} \right) \quad (19)$$

Where, n_L is the number of Landau levels. $\omega_c = \frac{eH}{mc}$ -cyclotron frequency.

It is known that in three-dimensional semiconductors, a continuous quadratic energy spectrum $\frac{p_z^2}{2m}$ is added to the energy spectrum of formula (20). However, in two-dimensional semiconductors, the movement of electrons along the Z-axis is quantized:

Indeed, the thickness of the quantum well d is covered by the dimensional quantization condition, in other words, the thickness is relatively close to the de Broglie wavelength of the electron in the crystal. The motion of an electron along the Z axis is calculated from the potential V_z :

$$V(z) = \begin{cases} 0, & 0 < z < d, \\ \infty, & z \leq 0, z \geq d \end{cases} \quad (21)$$

In the absence of a magnetic field in two-dimensional electron gases, the normalized wave functions of particles have the following form [13]:

$$\psi_{k_{fx}, k_{fy}, n_{fz}}(x, y, z) = \frac{1}{\sqrt{L_{f1}}} \exp(ik_{fx}x) \frac{1}{\sqrt{L_{f2}}} \exp(ik_{fy}y) \varphi_{n_z}(z) \quad (22)$$

Where, k_{fx} , k_{fy} are the wave numbers for the Fermi energy of electrons, n_{fz} is the number of dimensional quantizers along the Z axis.

In formula (9), the normalized functions $\varphi_{n_z}(z)$ in accordance with (8) are written in the following form:

$$\varphi_{n_z}(z) = \sqrt{\frac{2}{d}} \sin \frac{\pi n_z z}{d}, \quad n = 1, 2, 3, \dots \quad (23)$$

The Fermi energies of electrons corresponding to states (9) will be

$$E(k_{fx}, k_{fy}, n_{fz}) = \frac{\hbar^2}{2m} (k_{fx}^2 + k_{fy}^2) + \frac{\pi^2 \hbar^2 n_{fz}^2}{2md^2} \quad (24)$$

Substituting expressions (7), (11) into (6), we obtain the following formula in the presence of a magnetic field:

$$\mu_F(H) = \frac{\pi \hbar^2 N^{2d}(H)}{4mL^2} + \frac{\pi^2 \hbar^2 n_{fz}^2}{2md^2} \quad (25)$$

For an area equal to one ($L_x L_y = 1$) of formulas (12), the following is calculated:

$$\mu_F(H) = \frac{\pi \hbar^2 N^{2d}(H)}{4m} + \frac{\pi^2 \hbar^2 n_{fz}^2}{2md^2} = \frac{1}{8} \hbar \frac{eH}{mc} \cdot \frac{2\pi \hbar N^{2d}(H)c}{eH} + \frac{\pi^2 \hbar^2 n_{fz}^2}{2md^2} = \frac{1}{8} \hbar \omega_c \nu + \frac{\pi^2 \hbar^2 n_{fz}^2}{2md^2} \quad (26)$$

Here, $\nu = \frac{2\pi \hbar c N^{2d}(H)}{eH}$ – filling factor [14]. This is the number of Landau levels, taking

into account their spin splitting, in a quantizing magnetic field, at absolute zero temperature, completely filled with electrons. This dimensionless parameter is used for convenience in discussing quantum oscillatory effects in 2D electron gases.

As can be seen from formula (13), the Fermi energies are quantized if the filling factor is an integer, then the minimum energy quantum will be $\frac{1}{8} \hbar \omega_c$, that is, formula (13) gives the exact value of the energy for the first level corresponding to $\nu = 1$

$$\mu_1(H) = \frac{1}{8} \hbar \omega_c + \frac{\pi^2 \hbar^2 n_{fz}^2}{2md^2} \quad (27)$$

For all other levels, the rigorous theory gives the expression

$$\mu(H) = \hbar \omega_c \left(\nu + \frac{1}{8} \right) + \frac{\pi^2 \hbar^2 n_{fz}^2}{2md^2} \quad (28)$$

Here, the fill factor is an integer, $\nu = 0, 1, 2, 3, \dots$

In addition, in two-dimensional semiconductors, in the presence of a quantizing magnetic field, the energy spectrum of electrons is purely discrete. A purely discrete energy spectrum, in this case the Fermi energy, is usually characteristic of a quantum dot. In this case, the magnetic induction vector will be directed along the Z axis and perpendicularly along the plane of the transverse two-dimensional layer. In a transverse quantizing magnetic field, quantum wells become analogous to a quantum dot, in which motion is limited in all three directions.

RESULTS

It can be seen from the obtained formulas (15) that the Fermi energies depend strongly on the magnetic field, on the concentration of electrons, and on the thickness of the quantum well. In a quantizing magnetic field, the concentration of electrons in the considered two-dimensional semiconductors is determined in the following form [15]:

$$n^{2d} = \int_0^\infty \sum_{n_L} N_s^{2d}(E, H, n_L) f_0(E, E_F(H=0), T) dE \quad (29)$$

Here, $N_s^{2d}(E, H, n_L)$ - density of states of two-dimensional electronic systems when exposed to a quantizing magnetic field; $f_0(E, E_F(H=0), T)$ - Fermi-Dirac distribution function in the absence of a magnetic field.

In two-dimensional electronic systems, the energy density of states is taken as the sum of Gaussian peaks in the presence of a magnetic field, disregarding spin splittings [15]:

$$N_s^{2d}(E, H, n_L) = \frac{eH}{2\pi c} \sum_{n_L} \sqrt{\frac{2}{\pi}} \frac{1}{G} \exp \left[-2 \left(\frac{E - \hbar \omega_c \left(n_L + \frac{1}{2} \right)}{G} \right)^2 \right] \quad (30)$$

G is the broadening parameter taken constant. Here we consider two-dimensional electronic systems of noninteracting electrons according to the parabolic dispersion law at a finite temperature T , in the presence of a quantizing magnetic field B parallel to the growth direction.

And also, two features should be highlighted here. First, in addition to the Gaussian peak in the density of states, at each Landau level there is a common magnetic field factor B ahead of the total energy density of states. This means that with increasing magnetic field B , each Landau level can contain more and more electrons. Secondly, according to the form taken in the formula. According to (17), there is no density of states between Landau levels if their $\hbar \omega_c$ distance is noticeably larger than G .

Using expressions (15), (16), and (17), one can determine the dependence of the Fermi energy oscillations on the magnetic field, temperature, and thickness of the quantum well in two-dimensional semiconductors with a parabolic dispersion law without taking into account the spin per unit surface of the plane of motion:

$$\mu_F(H, T, d) = \frac{2\pi \hbar^2}{m} \int_0^\infty \sum_{n_L} N_s^{2d}(E, H, n_L) f_0(E, E_F(H=0), T) dE + \frac{\pi^2 \hbar^2 n_L^2}{2md^2} \quad (31)$$

Thus, using formula (18), one can calculate the dependence of the Fermi energy oscillations on the magnetic field, temperature, and thickness of a quantum well with a quadratic dispersion law. As can be seen from formula (18), oscillations of the density of energy states strongly affect the Fermi energies for two-dimensional electronic systems.

Let's analyze the Fermi energy oscillations for two-dimensional semiconductors. Fig.2 shows the dependence of the Fermi energy oscillations on the quantizing magnetic field for *InAs/GaSb/AlSb* quantum wells at a constant temperature and at a constant thickness of the quantum well. Here, temperatures are $T = 4.2\text{K}$, the thickness of the *InAs/GaSb/AlSb* quantum well is $d = 8\text{ nm}$, the number of Landau levels is $n_L = 10$, $G = 0.6\text{ meV}$, $E_F = 94\text{ meV}$ [16]. In this case, doped with Mn with a concentration of $5 \cdot 10^{16}\text{ cm}^{-3}$ on an *n-InAs* substrate and two quantum wells 12.5 nm in size (*InAs*) and 8 nm (*GaSb*) bounded by two *AlSb* barriers 30 nm thick [16]. As can be seen from the figure, with an increase in the magnetic field, the amplitude of the oscillation of the Fermi energy will increase.

$\mu_F(H, T, d)$ the graph (Fig.2) was constructed using formula (18). In addition, using formula (18), one can also obtain graphs $\mu_F(H, T, d)$ at different temperatures and at different thicknesses of quantum wells.

We now turn to calculating the dependence of the Fermi energy oscillations on the thickness d of the quantum well in a quantizing magnetic field with a parabolic dispersion law. We will be interested in the changes in the oscillation of the Fermi energy $\mu_F(H, T, d)$ at different d and at constant temperature. It is seen that the formula (18), $\mu_F(H, T, d)$ is inversely proportional to d^2 with another values constant. Fig.3 shows the oscillations of the Fermi energy in a quantizing magnetic field at different thicknesses of the quantum well d .

As can be seen from the figure, a decrease in the QW thickness d leads to an upward motion of the Fermi oscillations. Modern scientific literature indicates that in the absence of a magnetic field, the amplitude of the Fermi energy oscillations strongly depends on the thickness of the quantum well d .

But, as can be seen from Fig.2., the increase in amplitude depends only on the value of the magnetic field, and the thickness of the quantum well d leads to its motion along the axis $\mu_F(H, T, d)$.

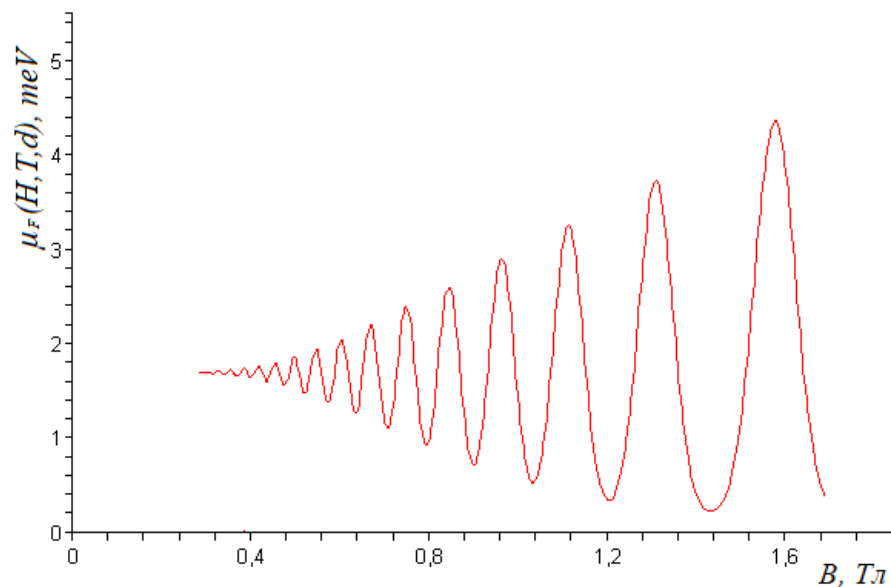


Fig.2. Dependence of the Fermi energy oscillations on the quantizing magnetic field with *InAs/GaSb/AlSb* quantum wells at $T=4.2$ K, $d=8$ nm. Calculated by the formula (18)

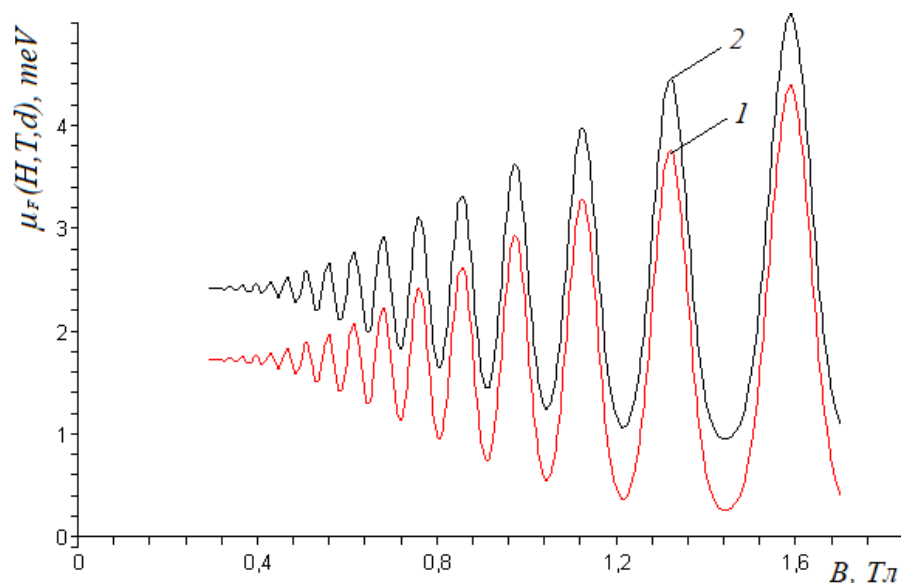


Fig.3. Influence of the thickness of the quantum well on the oscillations of the Fermi energy in a quantizing magnetic field. Here, $T = 4.2$ K, is calculated by formula (18) for *InAs/GaSb/AlSb* quantum wells.

1) $d=8$ nm, 2) $d=5$ nm.

DISCUSSION

In recent years, two-dimensional semiconductor materials have been the subject of intense theoretical and experimental studies and represent a dynamically developing field of semiconductor physics. The application of a strong magnetic field to two-dimensional semiconductor materials is a powerful tool for experimentally determining the basic parameters of the material, that is, their effective mass, Fermi energy and electron concentration. In quantizing magnetic fields, these parameters determine the relevance of experimental and theoretical studies of magneto-optical and electronic properties of nanoscale semiconductor devices and heterostructures based on them.

Now, let us analyze the oscillations of the Fermi energy of specific low-dimensional materials in a quantizing magnetic field. Fig.4 shows the oscillations of the Fermi energy when measuring $m=0.0665m_0$, $N=8 \cdot 10^{11} \text{ cm}^{-2}$, $G=0.5 \text{ meV}$ и $T=6 \text{ K}$ for two-dimensional electron gases in quantum wells (quantum wells, mainly *GaAs/GaAlAs* heterostructures) [15]. Let us calculate this graph of the quantized Fermi energy by $\mu_F(H, T, d)$ – functions. When calculating, take the ideal $\mu_F(H, T, d)$ by formula (18). A comparison of theory with experiment is shown in Fig.4 at various magnetic fields and constant temperatures. Using formulas (18), you can plot graphs $\mu_F(H, T, d)$ at high temperatures and at different thicknesses of the quantum well for quantum wells, mainly *GaAs/GaAlAs* heterostructures. It can be seen that the Fermi energy at a constant electron density is quantized rather strongly as a function of B in the theoretical and experimental plots in Fig.4.

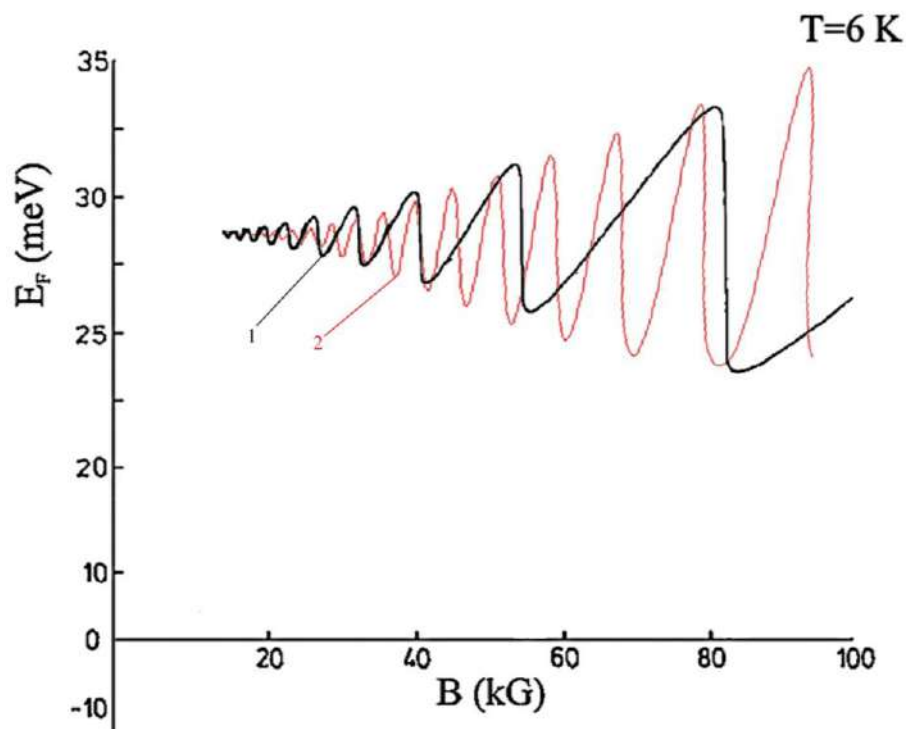


Fig.4. Oscillations of the Fermi energy when measuring $m=0.0665m_0$, $N=8 \cdot 10^{11} \text{ cm}^{-2}$, $G=0.5 \text{ meV}$ and $T=6 \text{ K}$ for two-dimensional electron gases in quantum wells (quantum wells, mainly *GaAs/GaAlAs* heterostructures). 1) experiment [15]; 2) theory calculated by formula (18)

CONCLUSION

Based on the study, the following conclusions can be drawn: It is shown that the Fermi levels of a nanoscale semiconductor in a quantizing magnetic field are quantized. A method is proposed for calculating the Fermi energy oscillations for a two-dimensional electron gas at different magnetic fields and temperatures. The proposed formula is used to study the experimental results in nanoscale semiconductor structures. Using formula (18), the Fermi energy oscillations are explained for two-dimensional electron gases in quantum wells (quantum wells, mainly *GaAs/GaAlAs* heterostructures) with a parabolic dispersion law.

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Муаллифлар ҳақида маълумот

Гулямов Гафур – физика-математика фанлари доктори, Наманган муҳандислик-қурилиш институти, “Физика” кафедраси профессори, gulyamov1949@mail.ru

Эркабоев Улугбек Инаятиллаевич – физика-математика фанлари доктори, Наманган муҳандислик-технология институти, “Технологик жараёнларни автоматлаштириш ва бошқариш ва информацион технологиялар” кафедраси мудири, профессор, erkaboev1983@mail.ru

Рахимов Рустамжон Ғуломжанович – Наманган муҳандислик-қурилиш институт докторанти, rgrakhimov@gmail.com

Сайидов Нозимжон Абдулносирович – Наманган муҳандислик-технология институти докторанти, sayidovnozimjon@gmail.com

Мирзаев Жасурбек Ибраилович – Наманган муҳандислик-технология институти, “Технологик жараёнларни автоматлаштириш ва бошқариш ва информацион технологиялар” кафедраси катта ўқитувчиси, jasmirfir@gmail.com

Телефон: +99893-052-38-83

MAGNETO-POLARITON EMISSION OF SEMICONDUCTORS CONSIDERING MECHANICAL EXCITON DECAY

I.I. Yulchiyev¹, B.J. Akhmadaliyev²

¹Doctoral student of Fergana State University, 150107, Fergana, Uzbekistan

²Fergana State Technical University, 150107, Fergana, Uzbekistan

Issued on July 30, 2025; accepted for publication on October 27, 2025

Abstract. This article investigates the magneto-optical properties of exciton–polaritons in semiconductor crystals with particular focus on the role of mechanical exciton decay. The study develops a theoretical framework describing magneto-spatial dispersion, critical decay phenomena, and interferometric emission, supported by experimental evidence. A central result is the identification of the decay parameter as a key factor governing polariton dynamics and coherence under magnetic fields. The findings highlight the potential for advanced applications such as highly spin-polarized polariton condensates, magnetically tunable polariton lasers, and compact magneto-optical devices, thereby positioning magneto-polariton systems as a promising platform for quantum photonic technologies.

Keywords: magneto-polariton emission, semiconductor crystals, exciton–polariton, mechanical exciton decay

Аннотация. В данной статье исследуются магнито-оптические свойства экситон–поляритонов в полупроводниковых кристаллах с учётом затухания механических экситонов. Разработана теоретическая модель, описывающая магнито-пространственную дисперсию, критические явления затухания и интерференционное излучение, что подтверждается экспериментальными данными. Показано, что параметр затухания играет ключевую роль в определении динамики поляритонов и когерентного излучения в магнитных полях. Полученные результаты открывают перспективы для создания новых фотонных устройств, включая сильно спин-поляризованные поляритонные конденсаты, магнитоуправляемые поляритонные лазеры и компактные магнито-оптические элементы.

Ключевые слова: магнито-поляритонное излучение, полупроводниковые кристаллы, экситон–поляритон, затухание механических экситонов

Annotatsiya. Ushbu maqolada yarimo'tkazgich kristallarda eksiton-polyaritonlarning magnito-optik xossalari mexanik eksitonlarni so'nish jarayonlarini xisobga olgan xolda chuqur tahlil qilingan. Tadqiqotda magnit-fazoviy dispersiya, kritik so'nish hodisalari va interferensiyalar nurlanish kabi murakkab kvanto-optik effektning nazariy modeli ishlab chiqildi hamda eksperimental natijalari bilan taqqoslandi. Asosiy e'tibor mexanik eksiton so'nish parametrining polariton dinamikasi va nurlanish jarayonlariga ta'sirini aniqlashga qaratilgan. Olingan natijalar magnit maydon ostida yuqori darajada boshqariladigan polyariton kondensatlari, polyariton lazerlari hamda yangi turdagi magnit-optik qurilmalarni yaratish imkonini beradi.

Kalit so'zlar: magnito-polyariton nurlanishi, yarimo'tkazgich kristallari, eksiton–polyariton, mexanik eksiton so'nishi

INTRODUCTION

The physics of exciton-polaritons in semiconductor crystals has evolved into a cornerstone of modern quantum optics and condensed matter physics, offering unprecedented opportunities for manipulating light-matter interactions at the quantum level [4][9]. In materials such as *CdS* and *CdSe*, the strong coupling between excitons (bound electron-hole pairs) and electromagnetic field modes gives rise to hybrid quasiparticles whose properties can be dramatically modified by

external magnetic fields and intrinsic decay mechanisms [1][2][12].

The mechanical exciton decay parameter $\hbar\Gamma$ emerges as a fundamental quantity that determines not only the lifetime of excitonic states but also the nature of polariton dynamics and the emergence of coherent optical phenomena [12]. This decay mechanism, which encompasses various scattering processes including exciton-phonon interactions, disorder-induced broadening, and radiative decay, becomes particularly significant when its magnitude approaches critical values that

fundamentally alter the light-matter coupling regime [1][2][3].

Recent theoretical and experimental advances have revealed that magneto-spatial dispersion effects in uniaxial crystals can lead to extraordinary magneto-optical phenomena, including polarization rotation without conventional Faraday effect, giant interferometric emission, and magnetically tunable coherent light sources [1][10][13]. These discoveries have opened new pathways for developing quantum photonic devices with unprecedented control over polarization states, coherence properties, and magneto-optical responses.

The critical insight underlying this work is the recognition that the parameter $\gamma_{LT} = \left(\frac{\omega_{LT}}{\Gamma}\right) \sqrt{\frac{M_{\parallel}}{M_{\perp}}} \sin^2 \theta$, which characterizes the effective light-exciton interaction strength, serves as a universal parameter governing the transition between different polariton regimes [2][3]. Near the critical value $\gamma_{LT} \approx 1$, the system exhibits remarkable phenomena including strong mode mixing, anomalous dispersion, and intense interferometric emission that challenges conventional understanding of polariton physics.

THEORETICAL FRAMEWORK AND MATHEMATICAL FORMALISM

The fundamental description of magneto-polariton phenomena requires a comprehensive treatment of the dielectric tensor in the presence of both magnetic fields and spatial dispersion. The complete expansion of the dielectric response function takes the form [1]:

$$\begin{aligned} \varepsilon_{\mu\nu}(\omega, \mathbf{k}, \mathbf{H}^{(0)}) = & \varepsilon_{\mu\nu}^{(0)}(\omega) + i\theta_{\mu\nu}(\omega)H_{\gamma}^{(0)} + \\ & + i\gamma_{\mu\nu}(\omega)k_{\gamma} + A_{\mu\nu\eta\gamma}(\omega)H_{\eta}^{(0)}k_{\gamma} + \\ & B_{\mu\nu\eta\gamma}(\omega)H_{\eta}^{(0)}k_{\gamma}k_{\zeta} + \dots \end{aligned} \quad (1)$$

The magneto-spatial dispersion tensor $A_{\mu\nu\eta\gamma}$ represents a fourth-rank tensor that couples magnetic field components with wave vector components, giving rise to novel magneto-

optical effects that cannot be described by conventional Faraday rotation or optical activity alone [1]. The symmetry requirements imposed by the Onsager relations for non-magnetic materials with time-reversal symmetry lead to fundamental constraints (2):

$$A_{\mu\nu\eta\gamma}(\omega) = A_{\mu\nu\gamma\eta}(\omega) = A_{\mu\nu\eta\gamma}^*(\omega) \quad (2)$$

For uniaxial crystals in the Voigt configuration ($\mathbf{H}^{(0)} \perp \mathbf{k}$), the magneto-spatial dispersion (MSD) tensor elements can be expressed in terms of material-specific parameters [1]:

$$A_{xx,zy} = A_{zz,xy} = \frac{\omega_{LT}^2 \Delta g}{2\omega_0(\omega_0^2 - \omega^2 + i\omega\Gamma)} \quad (3)$$

where Δg represents the effective g-factor anisotropy and ω_0 is the exciton resonance frequency.

The emergence of critical decay phenomena in polariton systems represents one of the most fascinating aspects of exciton-photon coupling physics. The critical decay value is determined by the condition where the discriminant of the polariton dispersion equation vanishes [2][3]:

$$\Gamma_c = \frac{\omega_{LT}}{\sqrt{\varepsilon_b}} \sqrt{\frac{M_{\parallel}}{M_{\perp}}} \cos \theta \quad (4)$$

where $M_{\parallel}$ and $M_{\perp}$ are the effective masses along parallel and perpendicular directions to the optical axis, respectively, and θ is the emission angle with respect to the crystal normal.

Near this critical value, the dispersion relation for mixed exciton-polariton modes develops a double root, leading to the formation of strongly mixed quantum states. The effective coupling

parameter $\gamma_{LT} = \left(\frac{\omega_{LT}}{\Gamma}\right) \sqrt{\frac{M_{\parallel}}{M_{\perp}}} \sin^2 \theta$ determines the nature of the polariton regime [2][3]:

Weak Coupling Regime ($\gamma_{LT} \ll 1$): In this regime, polariton branches are well separated, and the system exhibits conventional polariton behavior with Lorentzian spectral profiles and linewidth $\Delta \approx \hbar\Gamma$.

Intermediate Coupling Regime ($\gamma_{LT} \approx 1$): This critical regime is characterized by strong mode mixing, anomalous dispersion, and giant interferometric emission with asymmetric spectral profiles and complex angular dependencies.

Strong Coupling Regime ($\gamma_{LT} \gg 1$): Here, the system exhibits conventional Rabi splitting with well-defined upper and lower polariton branches.

The transition between these regimes can be controlled by varying the emission angle θ , which effectively modulates the longitudinal–transverse splitting according to

$$\omega_{\text{eff}} = \omega_{LT} \cos^2 \theta$$

Altogether, the advanced magneto-spatial dispersion theory combined with the analysis of critical decay phenomena and mixed-mode dynamics provides a unified framework for describing complex polariton behavior in semiconductor crystals.

BOLTZMANN KINETIC EQUATION, NON-EQUILIBRIUM DYNAMICS, AND MAGNETICALLY INDUCED COHERENCE

The dynamics of exciton-polaritons in the presence of strong decay and magnetic fields requires a sophisticated treatment based on the Boltzmann kinetic equation [14]. For polariton distribution functions $n^{(\pm)}(\omega, k)$, the kinetic equations take the form:

$$\frac{\partial n^{(\pm)}}{\partial t} + \mathbf{v}^{(\pm)} \cdot \nabla n^{(\pm)} = \left(\frac{\partial n^{(\pm)}}{\partial t} \right)_{\text{scatt}} + \left(\frac{\partial n^{(\pm)}}{\partial t} \right)_{\text{pump}} - \frac{n^{(\pm)}}{\tau^{(\pm)}} \quad (5)$$

where $\mathbf{v}^{(\pm)}$ are the group velocities of the polariton branches, $\tau^{(\pm)}$ are the branch-specific lifetimes, and the scattering terms include polariton–polariton, polariton–phonon, and polariton–exciton reservoir interactions [14].

In magnetic fields, the polariton energies are modified according to (6):

$$E^{(\pm)}(k, B) = E^{(\pm)}(k, 0) \pm 1/2 g_{\text{eff}} \mu_B B + \alpha_{\text{dia}} B^2 \quad (6)$$

where g_{eff} is the effective polariton g-factor and α_{dia} represents the diamagnetic coefficient [14]. One of the most remarkable consequences of the interplay between magnetic fields and critical decay is the emergence of magnetically controlled coherent emission. The coherence

function for polariton emission takes the form [6][11]:

$$g^{(2)}(\tau) = \frac{\langle I(t)I(t+\tau) \rangle}{\langle I(t) \rangle^2} \quad (7)$$

In the condensed phase, $g^{(2)}(0)$ approaches unity, indicating the establishment of quantum coherence. The coherence time τ_{coh} is inversely related to the effective decay:

$$\tau_{\text{coh}} = \frac{2\hbar}{\Gamma_{\text{eff}}(B, \theta)} \quad (8)$$

where Γ_{eff} includes both intrinsic decay and magnetic field-induced modifications.

EXPERIMENTAL SPECTROSCOPIC SIGNATURES AND MAGNETIC FIELD EFFECTS

Experimental investigations of magneto-polariton emission employ sophisticated spectroscopic techniques to probe the interplay between magnetic fields, spatial dispersion, and exciton decay. In CdSe crystals studied in Voigt configuration, transmission measurements reveal clear signatures of magneto-spatial dispersion effects [1][5]. The transmission coefficient for polarized light in the presence of MSD effects exhibits a characteristic angular dependence (9):

$$T_{\text{cross}} = \left| \frac{t_{xz}}{t_{zz}} \right|^2 = \left| \frac{f\Gamma}{\omega - \omega_{is} + i\Gamma} \right|^2 \quad (9)$$

where ω_{is} represents the isotropic frequency and f characterizes the MSD strength. The parameter f scales quadratically with magnetic field: $f^2 = (H^{(0)}/H_0)^2$, where $H_0 \approx 27.5 \text{ T}$ for CdSe [1]. Experimental measurements demonstrate polarization rotation angles reaching several degrees at moderate magnetic fields ($H \sim 6 \text{ T}$), confirming the theoretical predictions of MSD theory. The frequency dependence of the polarization rotation follows the resonant behavior characteristic of excitonic transitions [1].

Photoluminescence studies in mixed-mode geometry provide direct access to the critical decay phenomena and interferometric emission characteristics. Three distinct spectral regimes have been identified experimentally:

Regime I ($\Gamma < 0.3 \text{ meV}$): Conventional polariton emission with well-separated upper and lower branches, exhibiting standard Lorentzian lineshapes and weak angular dependence.

Regime II ($0.3 \text{ meV} \leq \Gamma \leq 1.0 \text{ meV}$): Transition regime characterized by the emergence of interferometric features, asymmetric spectral profiles, and strong angular dependence of emission characteristics.

Regime III ($\Gamma > 1.0 \text{ meV}$): Critical decay regime exhibiting giant interferometric emission, anomalous dispersion, and complex multi-peak spectral structures.

The partial intensities I_1 and I_2 corresponding to different polariton branches show remarkable behavior near critical decay:

$$I_{12} = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \varphi_{12}$$

where φ_{12} represents the phase difference between interfering modes. Near Γ_c , the interferometric term $2\sqrt{I_1 I_2} \cos \varphi_{12}$ can dominate the total emission intensity, leading to dramatic spectral modifications.

Recent experiments have also demonstrated the profound impact of magnetic fields on polariton condensation dynamics [16]. The condensation threshold power exhibits a complex dependence on magnetic field strength:

$$P_{th}(B) = P_{th}(0)[1 + \alpha B + \beta B^2]$$

where α and β are material-specific parameters that depend on the magnetic field orientation and polariton branch characteristics [16]. In semimagnetic semiconductors containing magnetic ions such as Mn^{2+} , giant Zeeman splittings enable highly spin-polarized polariton condensates even at moderate magnetic fields. Circular polarization degrees exceeding 95% have been achieved at $B = 3T$, representing a remarkable enhancement compared to non-magnetic systems [7][8].

ADVANCED APPLICATIONS OF MAGNETO-POLARITON SYSTEMS

The unique quantum properties of magneto-polariton systems create unprecedented opportunities for advanced

photonic and quantum information technologies. A key advantage arises from the high degree of spin polarization achievable in magnetic fields, which, combined with the long-range coherence of polariton condensates, enables the realization of novel quantum light sources for cryptographic and quantum information processing. The spin-orbit coupling in these systems can be controlled by external magnetic fields, giving rise to artificial gauge fields and topological quantum phases. The effective Hamiltonian describing spin-polarized polaritons in magnetic fields is given in equation (10):

$$H_{\text{eff}} = \frac{\hbar^2 k^2}{2m_{\text{pol}}} + \alpha_{SO}(\boldsymbol{\sigma} \times \mathbf{k}) \cdot \hat{\mathbf{z}} + 1/2 g_{\text{pol}} \mu_B \boldsymbol{\sigma} \cdot \mathbf{B} \quad (10)$$

where α_{SO} represents the spin-orbit coupling strength and σ are the Pauli matrices.

In addition to their role in quantum information, magneto-polariton systems also open pathways for revolutionary magneto-optical devices. The discovery of magneto-spatial dispersion effects allows for compact polarization rotators that operate efficiently over crystal-scale distances, unlike conventional Faraday devices which require longer propagation lengths. The performance of such devices is quantified by the figure of merit expressed in equation (11):

$$FOM = \frac{\theta_{\text{rot}}}{L \cdot H} = \frac{|f|}{L \cdot H_0} \cdot \frac{\Gamma}{\sqrt{(\omega - \omega_{is})^2 + \Gamma^2}} \quad (11)$$

where θ_{rot} is the rotation angle, L is the crystal thickness, and H is the applied magnetic field. Theoretical predictions suggest that optimized CdSe-based structures can achieve rotation angles of nearly one degree per micron per Tesla, representing an order of magnitude improvement compared to standard materials.

Furthermore, interferometric emission phenomena near the regime of critical decay provide a foundation for developing new types of coherent light sources. Unlike conventional lasers that rely on population inversion, these so-called “polariton lasers” are driven by stimulated scattering processes into coherent polariton modes. The threshold condition for polariton lasing, given by equation (12),

highlights the balance between scattering rates and loss processes:

$$R_{\text{stim}} \geq R_{\text{loss}} = \frac{1}{\tau_{\text{pol}}} + \frac{1}{\tau_{\text{cav}}} \quad (12)$$

where R_{stim} is the stimulated scattering rate, τ_{pol} is the polariton lifetime, and τ_{cav} is the cavity photon lifetime. Importantly, the magnetic tunability of polariton properties enables unprecedented control over both spectral and temporal characteristics of these coherent light sources, paving the way for electrically and magnetically tunable photonic devices.

Altogether, these advanced applications—spanning quantum information, magneto-optical components, and novel coherent light generation—position magneto-polariton systems as a transformative platform for next-generation quantum photonic technologies.

CONCLUSION

The study demonstrates that mechanical exciton decay is a decisive factor in shaping the magneto-optical properties of semiconductor polariton systems. Near the critical decay regime, strong interference effects, enhanced polarization control, and magnetically tunable coherent emission emerge, revealing new aspects of exciton–photon interactions. The theoretical framework developed provides a unified description of polariton behavior under magnetic fields, aligning well with experimental observations. These findings not only advance the fundamental understanding of magneto-polariton physics but also establish a foundation for innovative photonic applications, including highly efficient polarization rotators, spin-polarized polariton condensates, and magnetically tunable polariton lasers. Ultimately, magneto-polariton systems are shown to be a promising platform for future quantum photonic technologies.

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Information about the Authors

Yulchiyev Iftikhorjon Isaqovich - Doctoral student of Fergana State University, iftixorjon.yulchiyev@gmail.com, tel: +998505005880

Akhmadaliyev Bozorboy Jaboraliyevich - Doctor of Philosophy (PhD) in Physical and Mathematical Sciences, Associate Professor, Department of Physics, Fergana State Technical University, bozorboy.axmadaliyev@gmail.com tel: +998912825202

Fe₅Ga₆Gd₂YZr kristallidagi magnit tartiblanishiga temperaturaviy va magnit ta'sirlar.

Djurayev Davron Raxmonovich, Saidov Qurbon Sayfullayevich, Fayziyev Shaxobiddin Shavkatovich, Nasirova Nargiza Gayratovna
Buxoro davlat uiversiteti

2025-yil 2-iyunda berilgan; 2025-yil 28-oktyabrda chop etish uchun qabul qilingan.

Annotatsiya. Mazkur tadqiqot ishida Fe₅Ga₆Gd₂YZr kristalining strukturasini va kristall xossalari rentgen-fluoresans spektroskopiyasi natijalari va kombinatsion sochilish spektri natijalaridan o'rganildi. Spektrdagi optik o'tishlar (cho'qqilar)ni olish uchun Raman spektroskopiyasi va rentgen-fluoresensiya (XRF) tahlili qo'llanildi, mos ravishda strukturaviy xususiyatlar va elementar tarkibini o'rganish uchun foydalanildi. Kombinatsion sochilish va rentgen tahlil spektrlarida Fe, Ga, Gd, Y va Zr elementlarining xarakterli chiziqlari intensivligi olingan. Shuningdek, namunadagi domen strukturasini tartiblanishining temperaturaga bog'liqligi 134–296 K oralig'ida tadqiq qilindi. Tadqiqot shuni ko'rsatdiki, namunaning temperaturasining pasayishida domen strukturasida yuqori domen tartiblanishlar kuzatiladi. Temperaturaning oshishi bilan domen chegaralari orasidagi o'lchami kengayib borishi kuzatildi. Bundan tashqari, tashqi magnit maydonning domen tuzilishiga ta'siri 0–4,6 mT magnit maydoni oralig'ida tadqiq qilindi. Magnit maydonning ortishi va kamayib borish jarayonidagi domen struktura o'zgarishi tasvirlarining qiyosiy tahlili kristalda qoldiq magnit maydoni (magnitlanish) paydo bo'lganligi vizual kuzatildi.

Kalit so'zlar : kombinatsion sochilish spektri, rentgen-fluoresensiya tahlil, qoldiq magnit maydon, domen struktura, yuqori entropiyali qotishmalar.

Аннотация. В данной работе структура и кристаллические свойства кристалла Fe₅Ga₆Gd₂YZr были изучены по результатам рентгеновской флуоресцентной спектроскопии и спектров комбинаторного рассеяния. Рамановская спектроскопия и рентгеновская флуоресцентная (РФА) спектроскопия использовались для получения оптических переходов (пики) в спектре и были использованы для изучения структурных свойств и элементного состава, соответственно. Была получена интенсивность характеристических линий элементов Fe, Ga, Gd, Y и Zr в спектрах комбинаторного рассеяния и рентгеновского анализа. Также была изучена температурная зависимость упорядоченности доменной структуры в образце в диапазоне 134–296 K. Исследование показало, что с понижением температуры образца в доменной структуре наблюдается более высокий уровень упорядоченности доменов. С ростом температуры наблюдалось расширение размера между доменными границами. Кроме того, было изучено влияние внешнего магнитного поля на доменную структуру в диапазоне магнитного поля 0–4,6 мТл. Сравнительный анализ изображений изменения доменной структуры в процессе увеличения и уменьшения магнитного поля визуально наблюдал появление остаточного магнитного поля (намагниченности) в кристалле.

Ключевые слова: комбинаторный спектр рассеяния, рентгенофлуоресцентный анализ, остаточное магнитное поле, доменная структура, высокоэнтропийные сплавы.

Annotation. In this research, the structure and crystal properties of the Fe₅Ga₆Gd₂YZr crystal were investigated using X-ray fluorescence spectroscopy and Raman scattering spectra. Raman spectroscopy and X-ray fluorescence (XRF) analysis were employed to obtain optical transitions (peaks) in the spectra and to study the structural features and elemental composition, respectively. In the Raman and XRF spectra, characteristic lines corresponding to Fe, Ga, Gd, Y, and Zr elements were identified. Additionally, the temperature dependence of the domain structure ordering in the sample was studied within the range of 134–296 K. The results show that as the temperature decreases, a higher degree of domain ordering appears in the domain structure. With increasing temperature, an expansion of the distances between domain boundaries was observed. Furthermore, the effect of an external magnetic field on the domain configuration was examined in the range of 0–4.6 mT. A comparative analysis of the domain structure images obtained during the increase and decrease of the magnetic field visually revealed the presence of remanent magnetic fields (magnetization) in the crystal.

Keywords: Raman scattering spectrum, X-ray fluorescence analysis, remanent magnetic field, domain structure, high-entropy alloys.

Kirish

Yuqori entropiyali qotishmalar (YEQ) XXI asr boshlarida ilm-fanga kirib kelgan bo'lib, keyingi yillarda tadqiqotchilar va

muhandislarning katta qiziqishini uyg'otdi [1]. Ushbu qotishmalar yuqori konfiguratsion entropiyasi bilan ajralib turadi va shu sababli yuqori barqarorlik, keng mexanik, issiqlik

hamda funksional xususiyatlarni namoyon etadi. Mazkur materiallarning muhandislik sohalarida ko'p qirrali qo'llanilishi aynan shu fazilatlari bilan izohlanadi [2]. YEQLarga bo'lgan qiziqishning ortib borishi, birinchi navbatda, ularning yuqori mexanik mustahkamligi, yuqori temperaturalarda oksidlanishga qarshiligi hamda issiqlik barqarorligi bilan bog'liq. Ushbu noyob xususiyatlarning shakllanishiga yuqori entropiya effekti, diffuziya jarayonining sekinligi va kristall panjara buzilishlari asosiy omil bo'lib xizmat qiladi [3]. YEQLarning yuqori mustahkamlik, qattiqlik, strukturaviy barqarorlik, sozlanishi mumkin bo'lgan magnit fazilatlari hamda ko'p fazali strukturalarni hosil qilish qobiliyatiga egaligi ularni keng tadqiqot ob'ektiga aylantirmoqda [4]. Yuqori entropiyali qotishmalar ilk bor Yeh va hamkorlari tomonidan taklif etilgan bo'lib, ular besh yoki undan ortiq asosiy elementlarning deyarli ekvatomik nisbatlarda birikishi natijasida hosil bo'luvchi yangi turdagi qotishmalar sifatida ta'riflangan. Yuqori aralashish entropiyasi Gibbsning erkin energiyasini kamaytiradi va natijada murakkab intermetalik fazalardan ko'ra sodda qattiq eritma fazalari – masalan, yuz markazlashtirilgan, tanaga markazlashtirilgan yoki olti burchakli zich qadoqlangan kristall panjaralar shakllanishiga olib keladi [5]. Mazkur kontseptsiyaga asoslanib, yuqori temperatura sharoitida mustahkamlik, strukturaviy barqarorlik va oksidlanishga chidamlilikni oshirishga qaratilgan issiqlikka chidamli ko'p elementli optimallashtirilgan qotishmalar ishlab chiqilgan. Bunday qotishmalar ayniqsa aerokosmik texnologiyalar va energetika sanoatida muhim ahamiyat kasb etadi [6]. Shuningdek, tarkibida bir nechta asosiy elementlar mavjud bo'lishiga qaramay, YEQLar ko'pincha sodda bir fazali tuzilmalarni hosil qilishi mumkin [7]. Umuman olganda, ekvatomik yoki unga yaqin nisbatlarda tashkil topgan bir nechta asosiy elementlardan iborat yuqori entropiyali qotishmalar o'zlarining yuqori mexanik ko'rsatkichlari, issiqlik barqarorligi va korroziyaga chidamliligi tufayli materialshunoslikda istiqbolli yangi strukturaviy materiallar sifatida baholanmoqda [8]. Ularning noyob atomar tuzilishi, yuqori qattiqligi, egiluvchanligi, korroziyaga

qarshiligi va ekstremal sharoitlarda chidamliligi mazkur materiallarga bo'lgan ilmiy va amaliy qiziqishni yanada kuchaytirmoqda [9]. Shu jihatdan, YEQLar turli sanoat va strukturaviy qo'llanilishlar uchun istiqbolli nomzodlar sifatida ko'rilmoqda.

Jian Zhang va hammualliflar [10] CoCrFeNi yuqori entropiyali qotishmasini o'rganib, uning a'lo darajadagi mexanik xatti-harakati, korroziyaga chidamliligi va issiqlik barqarorligini ta'kidladilar. Ularning natijalari ushbu qotishmaning ilg'or ishlab chiqarish va muhandislik sohalarida katta salohiyatga ega ekanini ko'rsatdi. Brandon Wilfong va boshqalar [11] esa sintezdan keyingi tavlaniş temperaturaining yoyma eritilgan Fe_3Ga_4 namunalarining kristallografik va magnit xossalriga ta'sirini tadqiq etdilar. Ularning izlanishlari monokristall o'sishi va kuchlanishga bog'liq xatti-harakatlar, jumladan, nosimmetrik spin zichligi to'liqini (ISDW) fazasiga oid mexanizmlar haqida muhim ma'lumot berdi. Bu esa qurilma darajasida integratsiya uchun material xossalarni sozlash bo'yicha amaliy ko'rsatmalar taqdim etdi. Daiyi Deng va hamkorlari [12] ikki bosqichli qaritish (aging) jarayoniga duchor qilingan yuqori mustahkam va egiluvchan Mg-9.5Gd-2.3Y-1Zn-0.5Zr (VW92, wt%) qotishmasining mikrostrukturaviy evolyutsiyasini chuqur o'rgandilar. Tadqiqot natijalariga ko'ra, ikki cho'qqili qaritish hodisasi kuzatilgan bo'lib, birinchi cho'qqi 96, 48 va 48 soatda, ikkinchi cho'qqi esa mos ravishda 200, 180 va 184 soatlarda qayd etildi. Qarish davomiyligi ortishi faza o'zgarishlarini kuchaytirgani va uch o'lchamli struktura tarmog'ining shakllanishiga olib kelgani aniqlandi. J.M. Park va boshqalar [13] $\text{Fe}_{74-x}\text{Nb}_6\text{B}_{17}\text{Y}_3(\text{Zr},\text{Co})_x$ ($x = 3, 5, 8$) amorf qotishmalarida Fe elementini Zr yoki Co bilan almashtirish ta'sirini o'rganib, bunday elementar o'zgarishlarning magnit va mexanik xossalarga sezilarli ta'sir ko'rsatishini aniqladilar. J.H. Mendez va hammualliflar [14] Fe_3Ga_4 monokristallarining magnit, termodinamik va zaryad tashish xususiyatlarini, xususan, magnit holatlarning temperatura va tashqi magnit maydonga bog'liqligini tahlil qildilar. K.O. Moura va hamkorlari [15] esa Fe_3Ga_4 ning nanoyadro hosil qilishiga erishib, unda ferromagnetik-ferromagnetik va

ferromagnetik-antiferromagnetik o'tishlarning elektron tuzilma o'zgarishlari bilan bog'liqligini aniqladilar. L. Yang va boshqalar [16] Mg–10(Dy + Gd)–0.2Zr qotishmasining mexanik xossalari tarkibiy nisbat va issiqlik ishlov berish parametrlariga qarab samarali ravishda boshqarilishi mumkinligini ko'rsatdilar. O. Abounachit va hammualiflar [17] Co₂CrBi birikmasining elektron, struktura, optik, termodinamik va termoelektrik xossalarini, jumladan, ferromagnetik va antiferromagnetik fazalarini batafsil o'rgandilar. C. Aksu Canbay va boshqalar [18] yangi qotishma tarkiblarida faza o'zgarishlarini kuzatib, vaqt va temperatura parametrlariga bog'liqligini aniqladilar. Zeesham Abbas va hamkorlari [19] LiCoO₂ va LiNiO₂ birikmalarida magnit tartiblanishni tahlil qilib, ularning ferromagnetik xususiyatlarga egaligini tasdiqladilar. K. Meliani va boshqalar [20] bosim ta'sirida birikmalarning fizik xossalarida yuz beradigan o'zgarishlarni tadqiq qilib, ikki xil strukturaviy fazani aniqladilar. G. Surucu va hammualiflar [21] zichlik funksional nazariyasi (DFT) va umumlashtirilgan gradient yaqinlashuvi (GGA) asosida Pd₃X (X = Ti, Zr, Hf) intermetallik birikmalarning D0₂₄ va L1₂ tuzilmalaridagi struktura, mexanik, elektron va panjara dinamik xossalarini tahlil qildilar. Osvaldo F. Schilling [22] MnFeAs va MnFeAs_xP_{1-x} (0.25 ≤ x ≤ 0.65) birikmalarining almashinish konstantalari, kristallografik parametrlari va magnitlanish o'lchovlarini o'rgandi. Aeshah Alasmari va boshqalar [23] AC₂O₄ birikmalarida Ni va Fe kationlarini almashtirish magnit va dielektrik xossalarga ta'sirini tadqiq etdilar hamda 0 dan 10 mT gacha bo'lgan magnit maydonlarda dielektrik xatti-harakatni baholadilar. N.I. Abu Elsaad va hamkorlari [24] ErFeO₃ ortoferrit fazasiga Er³⁺ ionlari konsentratsiyasini (x = 0.075 gacha) kiritish mikrostrukturaviy parametrlarga qanday ta'sir qilishini ko'rsatdilar. Ular panjara konstantalari, rentgen zichligi, panjara deformatsiyasi, kristallit o'lchami va sirt maydoni bo'yicha tahlil olib bordilar. Jose Luis Hidalgo González [25] turli qalinlikdagi FeNi yupqa plyonkalarni sintez qilib, ularning magnit xossalarini izotermik qoldiq magnitlanish o'lchovlari orqali tahlil qildi. Shuningdek, plyonkalarda dipolyar maydonlarning gisterezis halqasiga ta'sirini

ham o'rgandi. Marcelo Salvador va boshqalar [26] magnitostatik energiyadagi bir o'qli anizotropiya va antiferromagnetik hissa o'rtasidagi o'zaro ta'sirni ko'rib chiqib, ularning raqobatli xatti-harakatini aniqladilar. O'rtacha zarracha oralig'ining ortishi dipolyar energiyani kamaytirib, zarracha magnit momentlarining asosan zanjir yo'nalishiga nisbatan qiyshaygan anizotropiya o'qiga yo'nalishini ko'rsatdi. Muhammad Zahid va hamkorlari [27] Ba_{1-x}Co_xFe₁₂O₁₉ tarkibidagi M-tipli geksaferritlarni muvaffaqiyatli sintez qilib, ularning struktura, morfologik, magnit va dielektrik xossalarini tavsifladilar. Luciano Nascimento va boshqalar [28] amorf Co₆₂Nb₃₂B₆ qotishmasining magnit parametrlarini o'rganib, uning xatti-harakati bo'yicha muhim natijalar taqdim etdilar. Pawan Chettr va hamkorlari [29] Fe bilan doplangan SnO₂ da antiferromagnetik holatning ferromagnetik holatga nisbatan barqarorroq ekanini ko'rsatib, Fe-modifikatsiyalangan qalay dioksid tuzilmalarining turli qo'llanmalar uchun samaradorligini tasdiqladilar. E. John Alex va hammualiflar [30] Bi_{1-x}Gd_xFeO₃ keramika yupqa plyonkalarini ishlab chiqib, ularning magneto-dielektrik xossalarini tahlil qildilar. V. Raghavan [31] tadqiqotlariga ko'ra, 500 °C da Fe–Ga–Gd tizimida GdFe₅Ga₇ va GdFe_{5.3}Ga_{6.7} kabi barqaror uchlamchi birikmalar hosil bo'lib, ular mos ravishda tetragonal va ortorombik simmetriyaga ega. Shu sababli 500 °C ushbu tizimdagi faza o'zgarishlarini o'rganish uchun muhim temperatura hisoblanadi. Simmetriya, faza tarkibi va kimyoviy nisbatlardagi o'zgarishlar magnit domen strukturasi shakllanishiga sezilarli ta'sir ko'rsatadi. Shuning uchun temperatura va tashqi magnit maydon ta'sirida domen xatti-harakatlarini tahlil qilishda faza barqarorligi va kristallografik simmetriya alohida e'tiborga olinishi kerak. P. Taheri [32] esa Galfenol (Fe–Ga) ni yengil nodir yer elementlari, jumladan Gd bilan doplash va termik ishlov berish uning magnit xossalari – magnetostriksiya, ko'rsatuvchanlik (coercivity) va to'yingan magnitlanishga sezilarli ta'sir ko'rsatishini isbotladi. Ularning natijalari issiqlik ishlov berish natijasida yuzaga keladigan mikrostrukturaviy o'zgarishlar tashqi magnit maydon ta'sirida magnit domen

strukturasini shakllantirishda hal qiluvchi rol o'ynashini ko'rsatadi.

Yuqori entropiyali qotishmalar (YEQ) materialshunoslikda inqilobiy kontsepsiya sifatida e'tirof etilib, ularning yuqori konfiguratsion entropiyasi, ko'p komponentli tuzilishi va o'ziga xos fizik xususiyatlari tufayli yangi imkoniyatlarni ochib bermoqda. Ushbu materiallar an'anaviy qotishmalarga xos ko'plab cheklovlarni bartaraf etib, ekstremal sharoitlarda qo'llash uchun katta ahamiyat kasb etadi. YEQlarni tadqiq etishning davomiy bosqichida murakkab birikma qotishma – $\text{Fe}_5\text{Ga}_6\text{Gd}_2\text{YZr}$ keyingi avlod magnit material sifatida o'rganildi. Ushbu tadqiqotda uning magnit holatlari, fazaviy o'tishlari va tashqi magnit maydon ta'sirining temperaturaga bog'liq ravishda o'zgarishi tizimli tarzda tahlil qilindi.

Materiallar va usullar

$\text{Fe}_5\text{Ga}_6\text{Gd}_2\text{YZr}$ birikmasining [1]o'rganilgan kristall tuzilishi murakkab ko'p

komponentli xususiyatga ega [33]. Ilmiy adabiyotlarda ushbu qotishmaning kristall simmetriyasi va fazaviy parametrlari bo'yicha to'liq ma'lumotlar mavjud emas. $\text{Fe}_5\text{Ga}_6\text{Gd}_2\text{YZr}$ kristallanish jarayonida ferrit-granat tipidagi yoki intermetallik fazalarga o'xshash tuzilmalarni hosil qilishi ehtimoldan xoli emas. Ammo aniq panjara parametrlari va simmetriya sinfi kompozitsiyaning tarkibi hamda sintez sharoitlariga bevosita bog'liqdir. Mazkur qotishmaning kristall tuzilishi haqida aniqroq tasavvurga ega bo'lish maqsadida namunalarning Raman spektroskopiyasi (2-rasm) hamda rentgen-fluoresensiya (XRF) (1-rasm) tahlillari o'tkazildi. Olingan XRF spektri va miqdoriy hisoblash natijalari (FP usuli asosida) fizik tahlil uchun asos bo'lib xizmat qildi. Rentgen nurlanishi ta'sirida namuna tarkibidagi atomlar qo'zg'alib, o'ziga xos fluoresensiya nurlanishini chiqaradi, bu esa elementlarning mavjudligi va ularning nisbatini aniqlash imkonini beradi

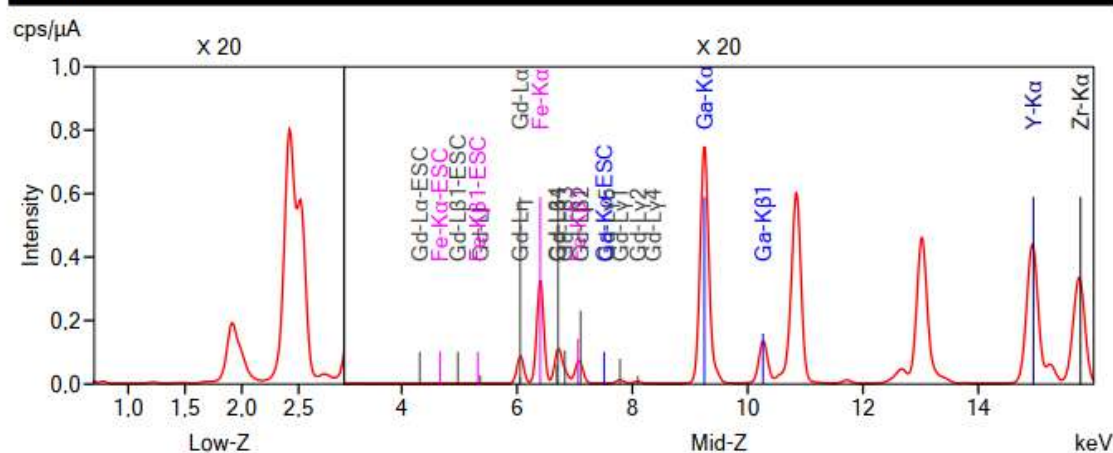
Meas. cond.

Diaphragm	DE-10mm	Atmosphere	Air	Sample spin	None	Meas. order optimization	Yes
Condition name	Primary filter	Tube voltage(kV)	35,0	Tube current(μA)	109	ST(μsec)	1,6
Mid-Z	C	35,0	109	1,6	100	Meas. time(sec)	37,3
Low-Z	Open	6,5	309	1,6	100	Dead time(%)	37,1

Analyzed result(FP method)

No.	Component	Result	Unit	LLD	LLQ	Element line	Intensity(cps/μA)
1	Fe	33,6	mol%	0,0083	0,0248	M:Fe-Kα	65,80590
2	Zr	6,90	mol%	0,0020	0,0061	M:Zr-Kα	106,22187
3	Ga	39,2	mol%	0,0076	0,0228	M:Ga-Kα	181,85389
4	Gd	10,7	mol%	0,0112	0,0337	M:Gd-Lα	19,92768
5	Y	9,67	mol%	0,0028	0,0083	M:Y-Kα	138,52889

Spectrum



1-rasm. RIGAKU NEX DE spektrometridagi $\text{Fe}_5\text{Ga}_6\text{Gd}_2\text{YZr}$ birikmasi namunasi va elementlarning rentgen-fluoresans spektrlari.

Rentgen-fluoresensiya spektrining tahlili (1-rasm) namunaning asosiy elementlari, kimyoviy tarkibi ($\text{Fe}_5\text{Ga}_6\text{Gd}_2\text{YZr}$ birikmasi) hamda spektral chiziqlarning o'ziga xos xususiyatlari haqida muhim ma'lumot beradi. O'rganilgan namuna murakkab ko'p komponentli qotishma bo'lib, gadoliniiy-temir-galliy asosida shakllangan va uning tarkibiga qo'shimcha ravishda ittriy hamda tsirkoniy elementlari kiritilgan. Kompozitsiyada asosiy rolni Fe, Ga va Gd elementlari bajaradi. Spekrni qayd etuvchi detektor ushbu elementlarning xarakterli rentgen emissiya chiziqlari (masalan, $K\alpha$, $L\alpha$ va sh.k.) intensivligini aniqlaydi.

Rentgen-fluoresans spektri asosida o'tkazilgan tahlillar namunaning asosiy element tarkibini hamda ularning spektral xususiyatlarini aniqlash imkonini berdi. O'rganilgan birikma ko'p komponentli qotishma bo'lib, uning asosiy elementlari sifatida temir, galliy va gadoliniiy ustunlik qiladi, shuningdek, tarkibida ittriy va tsirkoniy ham mavjud. Spektral tahlil natijalariga ko'ra, temirning miqdori 33,6 mol % ni tashkil etib, intensiv Fe $K\alpha$ chizig'i (6,4 keV) asosiy cho'qqi sifatida qayd etildi; shuningdek, taxminan 5,6 keV da Fe $K\alpha$ ESC escape-piki kuzatildi. Galliyning ulushi 39,2 mol % bo'lib, spektrda eng kuchli cho'qqi Ga $K\alpha$ (9,25 keV) da joylashgan, undan tashqari Ga $K\beta_1$ (10,26 keV) chizig'i hamda ~7,8 keV atrofida Ga $K\alpha$ escape-cho'qqisi aniqlandi. Gadoliniiy 10,7 mol % ni tashkil etib, uning L-seriyali chiziqlari – Gd $L\alpha$ (6,05 keV), Gd $L\beta_1$ (6,71 keV) va Gd $L\gamma_1$ (6,97 keV) qayd etildi; bu chiziqlar qisman Fe $K\alpha$ (6,4 keV) va Fe $K\beta$ (7,06 keV) bilan ustma-ust keladi. Namunaning tarkibida 9,67 mol % miqdorda ittriy mavjud bo'lib, spektrda aniq Y $K\alpha$ cho'qqisi (14,9 keV) qayd etildi. Tsirkoniy 6,90 mol % ulushga ega bo'lib, uning $K\alpha$ cho'qqisi (15,75 keV) Y $K\alpha$ chizig'iga yaqin joylashgani sababli ularning qisman qoplanishi kuzatildi. Umumiy hisob-kitobga ko'ra, namunadagi asosiy elementlar 99,97 mol % ni tashkil etib, sezilarli aralashmalar aniqlanmadi.

Spektral o'lchovlar jarayonida energiya diapazonlari, escape-piklarning mavjudligi hamda chiziqlar ustma-ust tushishi holatlari qayd etildi. Past energiyali diapazonda (0–1,5 keV) sezilarli cho'qqilar kuzatilmadi va bu

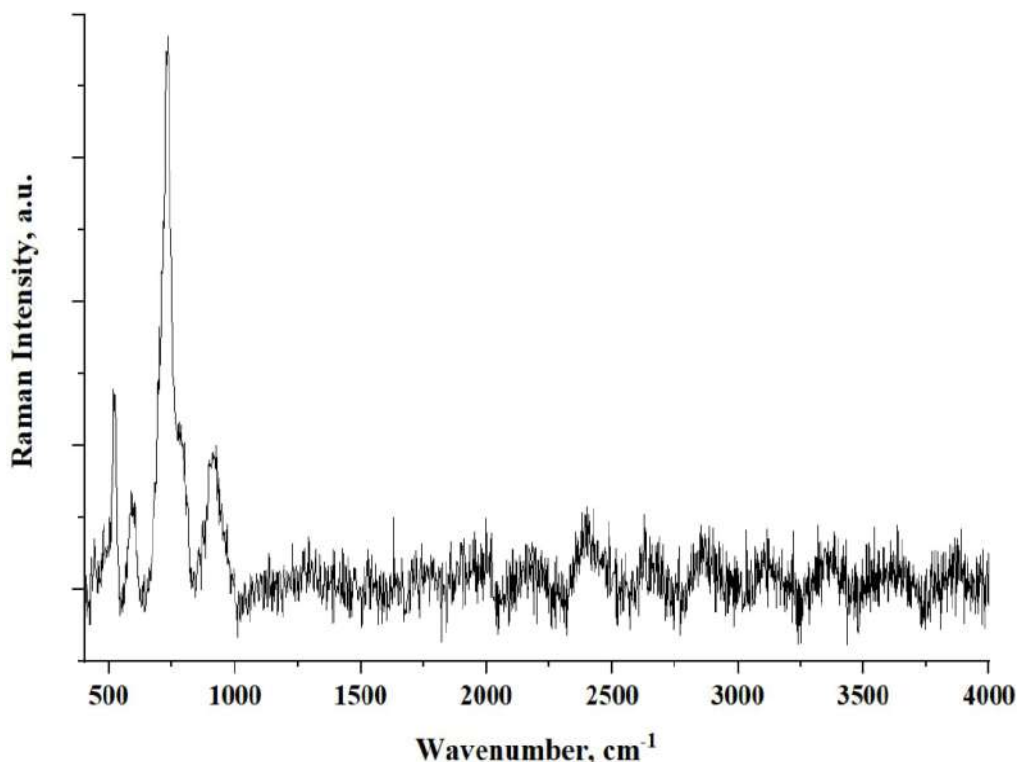
yengil elementlarning izlari yoki fon shovqini bilan bog'liq bo'lishi mumkin. O'rta energiya diapazonida (4–14 keV) asosiy elementlarning barcha xarakterli chiziqlari aniq qayd qilindi. Aniqlangan escape-piklar Gd $L\alpha$ (~4,5 keV), Fe $K\alpha$ (~5,6 keV) hamda Ga $K\alpha$ (~7,8 keV) ga to'g'ri keldi va bu detektorda energiya yutilish jarayonlari natijasida yuzaga kelgan. Gd L-seriyasi chiziqlarining Fe chiziqlari bilan qoplanishi FP (fundamental parametrlar) usuli yordamida matematik modellashirish orqali to'g'rilandi. Bundan tashqari, 0,5 keV dan pastda cho'qqilar kuzatilmaganligi namunaning tarkibida kislorod yo'qligini ko'rsatadi va bu metall qotishmalari uchun tabiiy holat hisoblanadi.

O'lchovlar ma'lum sharoitlarda amalga oshirildi: o'rta energiya diapazonidagi elementlarni (Fe, Ga, Gd) aniqlash uchun 35 kV kuchlanish va 109 μA tok ishlatildi, past energiyali diapazonda esa Bregg shovqinini kamaytirish maqsadida 6,5 kV kuchlanish va 309 μA tok qo'llanildi. Shuningdek, fon signallarini kamaytirish uchun "C" (uglerod) filtri, yengil elementlarga sezgirlikni oshirish uchun esa ochiq filtrdan foydalanildi. Havoning yumshoq rentgen nurlarini yutib yuborishi tufayli kislorod va uglerod kabi elementlarning spektral chiziqlari detektorda qayd etilmadi.

O'rganilgan spektr Fe–Ga–Gd elementlari ustunlik qiluvchi ko'p komponentli metall qotishmasiga mos keladi. FP tahlili natijalari eksperimental ma'lumotlar bilan yaxshi moslashgani ularning ishonchlilikini tasdiqlaydi. $\text{Fe}_5\text{Ga}_6\text{Gd}_2\text{YZr}$ murakkab intermetall birikma sifatida baholanishi mumkin bo'lib, uning kristall tuzilishini chuqurroq tahlil qilish orqali simmetriya o'qlarini aniqlash zarur. Ma'lumki, Fe_3Ga_4 birikmasi ortorombik kristall tizimga mansub bo'lib, yuz markazlashgan Bravais panjarasiga mos fazoda kristallanadi. Bunday tuzilma simmetriya elementlarining mavjudligi bilan tavsiflanadi: C_2 o'qi bo'ylab 180° aylanish, asosiy o'qlarga parallel bo'lgan tekisliklarda aks ettirish hamda inversiya markazi orqali elementlarning teskari tasviri. Bu kabi simmetriya elementlari ortorombik tizimning o'ziga xos xususiyatlari hisoblanadi va materialning fizik hamda kimyoviy xususiyatlariga bevosita ta'sir ko'rsatadi. Bundan tashqari, Gd, Y va Zr kabi qo'shimcha

elementlarning kiritilishi kristall panjaraning simmetriya xususiyatlarini o'zgartirishi mumkin. Biroq ularning struktura bo'yicha taqsimlanishi murakkab bo'lgani uchun simmetriyaga ta'sirini aniq baholash qiyin. Ortorombik tizim odatda C_2 , C_4 va C_6 simmetriya o'qlari bilan tavsiflanadi. Masalan, C_2 o'qi atrofida 180° , C_4 o'qi bo'yicha 90° va C_6 o'qi bo'yicha 60° aylanish simmetriyasi kuzatilishi mumkin. Ushbu simmetriya

elementlari Fe_3Ga_4 strukturasining o'ziga xosligi bilan bog'liq bo'lib, ular materialning tashqi ta'sirlarga, jumladan, magnit yoki elektr maydonlariga nisbatan xatti-harakatlarini tushinishda muhim ahamiyatga ega. Shu bois simmetriya o'qlarining chuqur tahlili yangi materiallarning fizik xususiyatlarini bashorat qilish va ular asosida kerakli parametrlarga ega qotishmalarni loyihalash uchun zaruriy shart hisoblanadi.



2-rasm. $Fe_3Ga_6Gd_2YZr$ namunasining Renishaw Invia Raman spektrometri yordamida spektral tahlili

Y va Zr elementlari bilan legirlangan Fe–Ga–Gd qotishmasining Raman spektr grafigi (2-rasm) asosida bir qator umumiy xulosalar chiqarish mumkin.

Asosiy cho'qqilar. Spekrda ~ 1500 – 3500 cm^{-1} diapazonida to'rtta aniq intensiv maksimum kuzatiladi. Ulardan eng yuqorisi $\sim 2500\text{ cm}^{-1}$ da joylashgan bo'lib, qolgan cho'qqilar taxminan $\sim 1800\text{ cm}^{-1}$, $\sim 2100\text{ cm}^{-1}$ va ~ 2900 – 3000 cm^{-1} oralig'ida aniqlangan.

Yuqori to'liq sonli diapazon (2800 – 4000 cm^{-1}). ~ 2900 – 3000 cm^{-1} sohasida qayd etilgan keng maksimum O–H valent tebranishlariga xos bo'lib, ko'pincha qotishma yuzasida oksidlanish jarayonlari yoki namlikning yutilishi bilan bog'liq bo'lishi

mumkin. 3000 cm^{-1} dan yuqori chastotalarda aniq cho'qqilarning yo'qligi tuzilishda C–H bog'lari yoki organik aralashmalar sezilarli darajada mavjud emasligini ko'rsatadi.

O'rta chastotalar sohasi (1500 – 2800 cm^{-1}).

$\sim 1800\text{ cm}^{-1}$, $\sim 2100\text{ cm}^{-1}$ va $\sim 2500\text{ cm}^{-1}$ dagi signallar eng qiziqarli diapazonni tashkil etadi. Ular sof metallarning asosiy fonon tebranishlariga to'g'ridan-to'g'ri mos kelmaydi, chunki Fe, Ga, Gd, Y va Zr elementlarida Raman faol rejimlar odatda juda zaif yoki aniqlanmaydi. Ushbu cho'qqilar qotishmada oksid fazalari yoki intermetallik tuzilmalar shakllanishi bilan bog'liq bo'lishi mumkin:

Oksid fazalari. 1800–2500 sm^{-1} oralig'ida kuzatilgan chiziqlar galiy (Ga_2O_3) yoki nodir yer oksidlari (Gd_2O_3 , Y_2O_3) ga mos kelishi ehtimoldan xoli emas. ZrO_2 uchun esa asosiy Raman tebranishlari odatda past chastota mintaqasida ($<1000 \text{ sm}^{-1}$) kuzatiladi.

Intermetall birikmalar. Qayd etilgan cho'qqilar Fe–Ga–Gd–Y–Zr tizimida hosil bo'ladigan murakkab intermetallik fazalarning tebranish rejimlarini aks ettirishi mumkin. Ularni aniq identifikatsiya qilish uchun mavjud spektral ma'lumotlar bazalari bilan solishtirish zarur.

Uglerodli aralashmalar. $\sim 2500 \text{ sm}^{-1}$ dagi cho'qqi grafitning nuqsonli modifikatsiyalariga (D yoki D+D" rejimlari) mos kelishi mumkin. Bu esa namunani tayyorlash yoki tashqi muhit ta'siri natijasida uglerod bilan ifloslanishni ko'rsatadi.

Past chastotalar sohasi (500–1500 sm^{-1}).

Ta'kidlash joizki, keltirilgan grafik 1500 sm^{-1} dan past chastotali sohani qamrab olmaydi. Biroq aynan shu diapazon oksid fazalarini aniqlashda muhim hisoblanadi, chunki Ga_2O_3 , Gd_2O_3 , Y_2O_3 va ZrO_2 oksidlarining asosiy intensiv Raman cho'qqilari odatda 1000 sm^{-1} dan pastda joylashadi.

Shu sohada, *shuningdek*:

- intermetallik fazalarning panjara tebranishlari;

- uglerodli materiallarga xos modlar, jumladan grafit/grafen uchun D-pik ($\sim 1350 \text{ sm}^{-1}$) va G-pik ($\sim 1580 \text{ sm}^{-1}$) ham joylashadi.

Olingan Raman sochilish ma'lumotlari asosida quyidagi dastlabki xulosalarni chiqarish mumkin:

Aniqlangan tebranish rejimlari. Spektrda ~ 495 , 615, 845 va 970 sm^{-1} atrofida qayd etilgan asosiy cho'qqilar ehtimol Fe–Ga bog'lanmalari hamda noyob yer elementlarini o'z ichiga olgan strukturaviy birliklarning (masalan, Gd–Fe yoki Y–O) panjara tebranishlaridan kelib chiqqan bo'lishi mumkin. Biroq, $\text{Fe}_3\text{Ga}_6\text{Gd}_2\text{YZr}$ tarkibiga oid aniq ma'lumotlar bazasining mavjud emasligi sababli fazalarni aniq identifikatsiya qilish hozircha imkonsizdir.

Yuza bilan bog'liq xususiyatlar. $\sim 3000 \text{ sm}^{-1}$ atrofida kuzatilgan keng signal sirt

gidroksil guruhlar yoki adsorbsiyalangan suv molekularining valent tebranishlariga mos keladi, bu esa namunada qisman yuzaki oksidlanish jarayoni sodir bo'lganidan darak beradi.

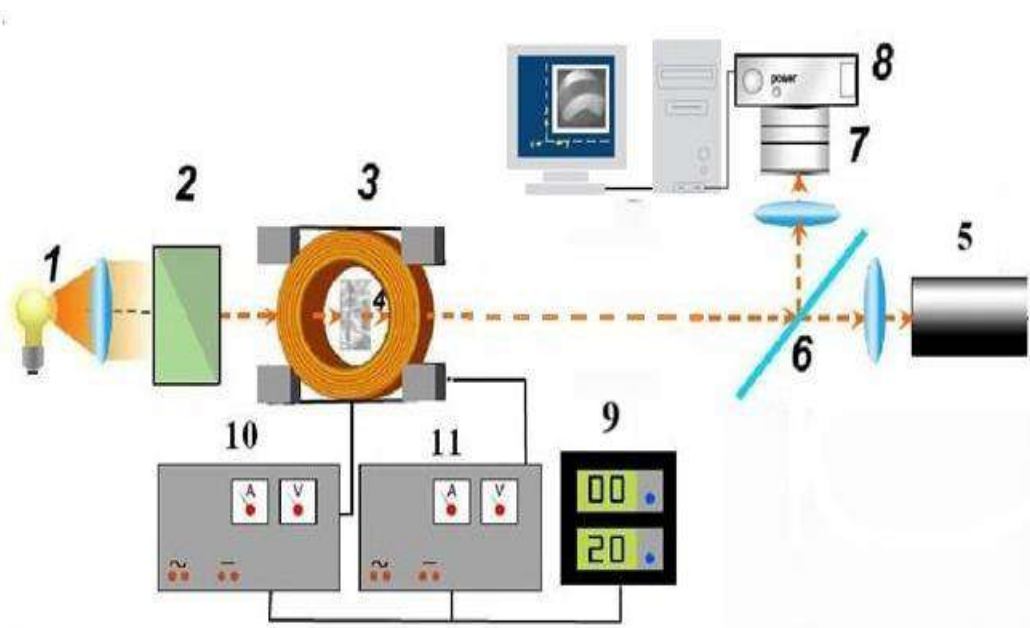
Kichik miqdordagi ifloslanishlar. $\sim 2500 \text{ sm}^{-1}$ dagi yakkalangan cho'qqi ehtimol uglerodli modifikatsiyalar yoki tashqi muhit ta'siri natijasida paydo bo'lgan ifloslanish bilan bog'liq bo'lishi mumkin. Buni ishonchli tasdiqlash uchun qo'shimcha yuzaki tahlil metodlaridan foydalanish talab qilinadi.

Domen tuzilmasini vizual tadqiq qilish uchun qurilma.

Tadqiqotlar «Metam P-1» metallografik mikroskopining optik tizimi yordamida amalga oshirildi. Yorug'lik manbai sifatida KGM12–100 rusumli galogen lampasi (1) qo'llanilib, undan chiqayotgan oqim linza (2) orqali parallel nur to'plamiga aylantirildi. Keyinchalik u polyaryzator (3) orqali o'tkazilib, tadqiq qilinayotgan namuna (4) yuzasiga yo'naltirildi va hosil bo'lgan nur mikroskop ob'ektiviga (5) kiritildi. Namuna optik kriostat ichida joylashtirildi va u o'zaro perpendikulyar joylashgan ikkita magnit g'altak (10) orasiga o'rnatildi. Ushbu g'altaklar yordamida $B \leq 4,6 \text{ mT}$ gacha bo'lgan bir jinsli magnit maydon hosil qilish, shuningdek, magnit induksiya vektorini optik o'qqa nisbatan $\pm 2^\circ$ aniqlikda yo'naltirish imkoniyati yaratildi ($|B| = \text{const}$ sharoitida).

Fe–Ga–Gd kristallining Y va Zr bilan legirlangan namunasi domen tuzilmasiga temperaturaning ta'sirini o'rganish

M.Z. Sharipov va hammualliflar [34] tomonidan terbiy temir oksidi kristallarida domen tuzilishining magnit kompensatsiya harorati yaqinidagi xatti-harakati tadqiq qilingan. Tadqiqot natijalariga ko'ra, haroratning o'zgarishi domen chegaralarining qayta tuzilishiga, ularning yo'nalishidagi o'zgarishlarga hamda magnit-optik xossalarga sezilarli ta'sir ko'rsatishi aniqlangan. Ushbu natijalar shuni ko'rsatadiki, issiqlik va magnit sharoitlaridagi nozik o'zgarishlar kristallning domen tashkilotiga kuchli ta'sir qilishi mumkin. Bu holat Fe–Ga–Gd tizimi uchun ham dolzarb hisoblanadi.

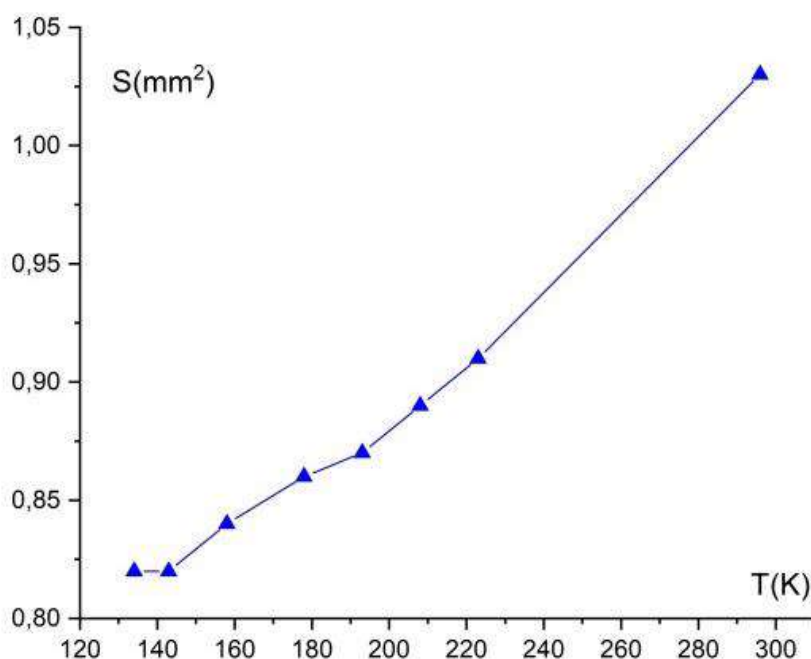


3-rasm. Tajriba qurilmasining blok-sxemasi. 1 – yorug‘lik manbai (KGM12–100 galogen lampasi); 2 – polyaryzator; 3 – Helmholtz g‘altaklari; 4 – namuna ($\text{Fe}_5\text{Ga}_6\text{Gd}_2\text{YZr}$); 5– «Metam P-1» metallografik mikroskopi; 6- 7 – mikrofotoqrafik adapter; 8 – raqamli kamera; 9 – Helmholtz g‘altaklarini boshqaruvchi elektron quvvat bloki; 10, 11 – doimiy tok manbalari.

Jadval 1. $\text{Fe}_5\text{Ga}_6\text{Gd}_2\text{YZr}$ kristali uchun temperaturaning domen tuzilishiga ta'siri.

T, K	Domen tuzilishi tasvirlari		
296, 228, 208			
193, 178, 158			
143, 138, 134			

Y va Zr bilan legirlangan Fe–Ga–Gd kristallining domen tuzilishiga haroratning ta'siri. $\text{Fe}_5\text{Ga}_6\text{Gd}_2\text{YZr}$ birikmasining domen tuzilishi 134–296 K harorat oralig'ida o'rganildi. Haroratga bog'liq tajribalarni o'tkazish uchun namuna suyuq azotli kriostat ichiga joylashtirildi. Olingan kuzatuvlar shuni ko'rsatdiki, domen konfiguratsiyasi haroratning o'zgarishiga sezgir bo'lib, bu jarayon 1-jadvalda keltirilgan namunaviy tasvirlarda aniq ko'rinadi. Vizual tahlil natijalariga ko'ra, harorat pasayishi bilan domenlarning o'lchami ortib, ular bir-biri bilan qo'shilib borishi kuzatildi. Bu esa sovitish jarayonida namunaning magnit xossalari kuchayib borishini ko'rsatadi. Shu bilan birga, ayrim hollarda magnit tartiblanish turiga qarab teskarisi ham qayd etildi — ya'ni, sovitishda domenlarning kichrayishi. Kristall domenlaridagi qizil va och (sariq) qismlarning maydonining haroratga bog'liqligi 4-rasmda keltirilgan. Ma'lumotlar shuni ko'rsatadiki, domen maydonining haroratga bog'liqligi nolinear xarakterga ega. Bunday xulq-atvor domen tuzilishining sinkron emasligi bilan izohlanadi. Tasvirlar domenlarning harorat bo'yicha evolyutsiyasini yaqqol namoyish etadi. 134–296 K diapazonda domenlarning o'lchami va morfologiyasi sezilarli darajada o'zgaradi, bu esa kristallning magnit xossalari haroratga yuqori sezgir ekanini tasdiqlaydi.



4-rasm. $\text{Fe}_5\text{Ga}_6\text{Gd}_2\text{YZr}$ kristali uchun temperaturaning domen tuzilishiga ta'siri grafigi.

Tashqi magnit maydonning $\text{Fe}_5\text{Ga}_6\text{Gd}_2\text{YZr}$ kristallining domen tuzilmasiga ta'siri

Tajriba 26 °C temperaturada olib borildi. Namuna 0–4.6 mTl oralig'ida magnit maydonga joylashtirildi, maydon qiymati o'rtacha 0.12 mTl qadam bilan o'zgartirildi (2-jadval).

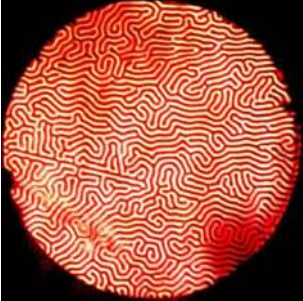
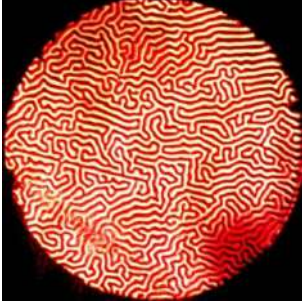
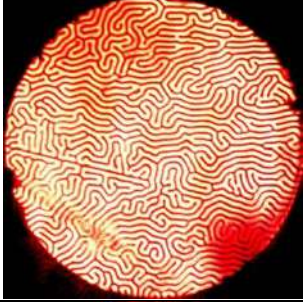
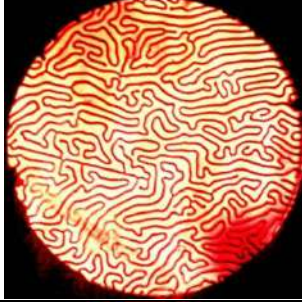
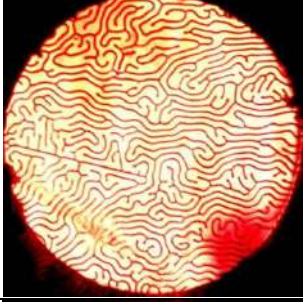
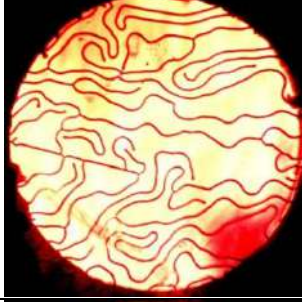
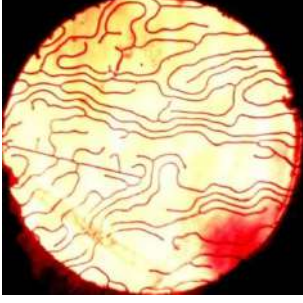
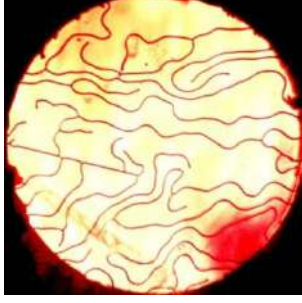
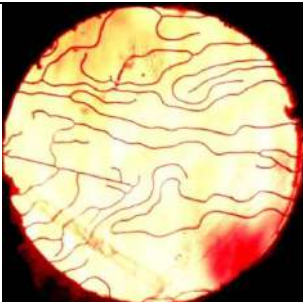
2.8 mTl dan boshlab domenlarning magnit tartiblanishida sezilarli o'zgarishlar kuzatildi. Maydonning yanada oshirilishi monodomenli tuzilishga intilish bilan kechadi.

Namunaning magnit gisterezini domenlarning tasvirlaridagi o'zgarishlar orqali vizual aniqlash mumkin. Jadvaldagi har bir magnit maydoni qiymatiga ikki xil domen tuzilmasi mos keladi — bu qiymatlar maydonni oshirish va kamaytirish yo'nalishlarida farq qilmaydi. Bu esa namunada qoluvchi qoldiq magnit maydon mavjudligini tushuntiradi.

Dzhuraev va boshqalar [35] tomonidan olib borilgan tadqiqotlarda $\text{FeBO}_3\text{:Mg}$ monokristallarida fazoviy modulyatsiyalangan magnit tartibning tashqi nurlanishga yuqori sezgirliги ko'rsatib berilgan. Fotoinduksiya natijasida domen konfiguratsiyasida mahalliy qayta

tuzilishlar kuzatilgan bo'lib, bu jarayon Fe–Ga–Gd tizimidagi issiqlik va magnit ta'sirlar ostida domenlarning qayta tashkil topish mexanizmlariga o'xshashdir. Bu esa tashqi omillar ta'sirida magnit tuzilmalarning qayta konfiguratsiyalanish mexanizmlari umumiy tabiatga ega ekanini tasdiqlaydi.

Jadval 2. Fe₅Ga₆Gd₂YZr kristali uchun magnit maydonning domen tuzilishiga ta'siri.

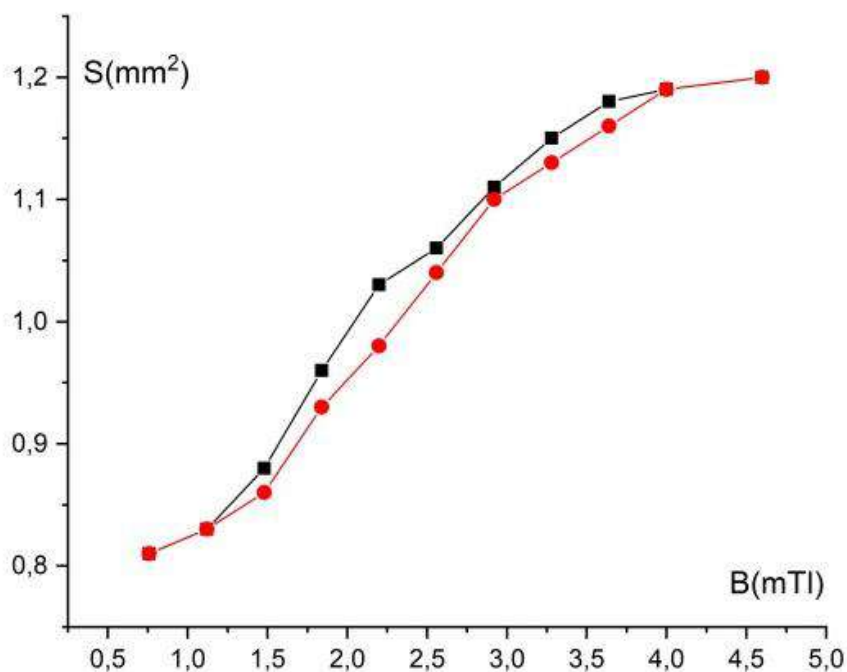
B, mTl	O'sib boorish	Qaytish
0		
1.84		
2.92		
4.00		
4.60		

Mazkur tajriba xona haroratida (26 °C) amalga oshirildi. Namuna 0 dan 4,6 mT gacha tashqi magnit maydonga ta'sir ettirildi, bunda maydon bosqichma-bosqich ~0,12 mT ga oshirib borildi

(5-rasm). Domenlarning magnit tartiblanishida sezilarli o'zgarishlar 2,8 mT dan boshlab kuzatildi. Maydonning yanada ortishi domenlarning bir-domenli konfiguratsiyaga intilishini yuzaga keltirdi. Namunadagi magnit gisterezis domen tuzilmalarining fotosuratlarini asosida vizual kuzatuv yo'li bilan aniqlangan. 2-jadvalda keltirilgan har bir magnit maydon qiymati uchun ikki xil domen tasvirlari keltirilgan bo'lib, ular mos ravishda tashqi maydonning ortishi va kamayishi jarayonlariga to'g'ri keladi. Ushbu gisterezisga o'xshash hodisa namunada qoldiq magnitlanish mavjudligini ko'rsatadi. Domen strukturalarining maydon (magnit) bog'liqligi 5-rasmda keltirilgan. Bu yerda yaqqol o'zgarishlar kuzatilgan bir nechta pozitsiyalar alohida ko'rib chiqildi. 5-rasmdan ko'rinib turibdiki, magnitlanish va demagnitlanish jarayonlari sezilarli darajada farq qiladi, bu esa namunada qoldiq magnit maydonning mavjudligini izohlaydi. Shuningdek, maydon qiymatlariga mos ravishda olingan holatlar sonini ko'paytirish orqali bu hodisa yanada ravshan namoyon bo'ladi.

Magnitlanish vektorini tarkibiy domenlarning magnitlanish vektorlari yig'indisi sifatida ifodalash mumkin, bunda domenlar o'zaro yaqin burchaklar α_i ostida yo'nalgan bo'ladi.

$$\vec{M} = \sum_i^n \vec{M}_i \cos \alpha_i$$



5-rasm. $Fe_5Ga_6Gd_2YZr$ kristali uchun magnit maydonning domen tuzilishiga ta'siri grafigi

Xulosa

Y va Zr elementlarining qo'shilishi Fe–Ga–Gd tizimining kristall panjara simmetriyasiga sezilarli ta'sir ko'rsatishi, tashqi magnit maydon ostida domen tuzilmasining xatti-harakatlaridan yaqqol namoyon bo'lmoqda.

Raman sochilish spektrlari oksid fazalarining (Ga, Gd, Y) yoki murakkab intermetall birikmalarining mavjudligini ko'rsatadi. Fe, Ga, Gd, Y va Zr ning sof metall

fazalari esa kuzatilgan spektral diapazonda sezilarli Raman cho'qqilarini bermaydi.

Y va Zr bilan legirlangan Fe–Ga–Gd qotishmasida domen tuzilmasining temperaturaga bog'liqligi shuni ko'rsatadiki, sovitish jarayonida atomlarning magnit momentlari orasidagi kuchli almashinish o'zaro ta'sirlari issiqlik tebranishlarining tartibsizlantiruvchi ta'siridan ustun keladi. Domenlarning turlicha yo'nalganligi esa domen tuzilmalarining temperaturaga bog'liq

bo'lgan xatti-harakatida nolinearlikka olib keladi.

Fe–Ga–Gd qotishmasining magnit maydonga bog'liq holda domen tuzilmasi esa namunadagi magnit domenlarning monodomen holatiga o'tishini ko'rsatadi. Bundan tashqari, bu jarayon orqali namunaning qoldiq magnit maydonini vizual aniqlash va domen sohalarining tashqi magnit maydonga bog'liqligini olish imkonini beradi.

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KOMPENSATSIYALANGAN Si<B,S> VA Si<B,Rh> MONOKRISTALLARIDA ELEKTR O'TKAZUVCHANLIKNING UZOQ MUDDATLI RELAKSATSIYA JARAYONLARI

A.Y. Boboyev¹, Z.M. Ibrohimov², A.K. Rafikov²

¹Z.M. Bobur nomidagi Andijon davlat universiteti, 1700100, Andijon, O'zbekiston

²Andijon davlat texnika instituti, Andijon, O'zbekiston

³O'zRFA Yadro Fizikasi Instituti, Toshkent, O'zbekiston

2025-yil 2-avgustda berilgan; 2025-yil 25-oktyabrda chop etish uchun qabul qilingan

Annotatsiya Maqolada Si<B,S> va Si<B,Rh> monokristallarida elektr o'tkazuvchanlikning relaksatsiya jarayonlari har xil konsentratsiyalardagi kompensatsiyalash sharoitida tadqiq qilingan. Fotoo'tkazuvchanlikning relaksatsiya jarayoni kompensatsiya qilingan Si<B,S> va Si<B,Rh> monokristallarida tezkor (τ_1) hamda sekin (τ_2) xarakterli vaqtlarga ega ikki bosqichli eksponensial qonuniyat bilan tavsiflanishi hamda bu relaksatsiya jarayonlari kompensatsiya qiluvchi kirishmaning turi va uning konsentratsiyasiga bog'liq ekanligi aniqlandi. Ryelaksatsiya parametrlari (τ_1 , τ_2) aniqlanib, Si<B,Rh> namunasida fototok relaksatsiyasining xarakterli vaqti Si<B,S> namunasiga nisbatan ancha qisqa ekanligi aniqlandi. γ -nurlanish dozasi oshib borishi bilan relaksatsiyaning ikkinchi xarakterli vaqti (τ_2) dastlab keskin ortadi, keyin esa ma'lum bir yuqori dozada to'yinish holatiga yetishi aniqlandi va bu holat nurlanish ta'siri natijasida hosil bo'ladigan chuqur energetik defektlar sonining cheklanganligi bilan izohlandi. Ryelaksatsiya vaqti (τ_2)ning γ -nurlanish flyuyensiga bog'liq o'zgarishi harorat pasayganida (77 K) yanada kuchayishi aniqlandi. Zaryad tashuvchilarning konsentratsiyasidagi fluktuatsiyalarning relaksatsiya jarayoniga ta'siri o'rganilib, boshlang'ich materialning solishtirma qarshiligi kamayganda, ya'ni yuqori konsentratsiyalarda fluktuatsiya amplitudasi oshib borishi va bu holatning relaksatsiya vaqtini uzaytirishi aniqlandi.

Kalit so'zlar: Plyonka; Si<B,S>; Si<B,Rh>; γ -nurlanish; fotoo'tkazuvchanlik; relaksatsiya vaqti; harorat; konsentratsiyalar

Аннотация В статье исследованы процессы релаксации электропроводности в монокристаллах Si<B,S> и Si<B,Rh> при различных условиях компенсации и концентрациях. Установлено, что процесс релаксации фотопроводимости в компенсированных монокристаллах Si<B,S> и Si<B,Rh> описывается двухступенчатой экспоненциальной зависимостью с характерными временами быстрой (τ_1) и медленной (τ_2) релаксации, причем эти процессы релаксации зависят от типа компенсирующей примеси и её концентрации. Определены параметры релаксации (τ_1 , τ_2), и установлено, что характерное время релаксации фототока в образце Si<B,Rh> значительно короче по сравнению с образцом Si<B,S>. При увеличении дозы γ -облучения время второй характерной релаксации (τ_2) сначала резко увеличивается, а затем достигает состояния насыщения при определенной высокой дозе, что объясняется ограниченным количеством глубоких энергетических дефектов, образующихся под воздействием облучения. Зависимость времени релаксации (τ_2) от флюенса γ -излучения усиливается при снижении температуры (до 77 K). Исследовано влияние флуктуаций концентрации носителей заряда на процесс релаксации, и установлено, что с уменьшением удельного сопротивления исходного материала, то есть при более высоких концентрациях, амплитуда флуктуаций увеличивается, что приводит к увеличению времени релаксации.

Ключевые слова: Пленка; Si<B,S>; Si<B,Rh>; γ -излучение; фотопроводимость; время релаксации; температура; концентрации

Abstract The article investigates the relaxation processes of electrical conductivity in Si<B,S> and Si<B,Rh> single crystals under various compensation conditions and concentrations. It is established that the relaxation process of photoconductivity in compensated Si<B,S> and Si<B,Rh> single crystals is described by a two-stage exponential dependence with characteristic times of fast (τ_1) and slow (τ_2) relaxation, and these relaxation processes depend on the type and concentration of the compensating impurity. The relaxation parameters (τ_1 , τ_2) are determined, and it is shown that the characteristic relaxation time of the photocurrent in the Si<B,Rh> sample is significantly shorter compared to the Si<B,S> sample. With increasing γ -irradiation dose, the second characteristic relaxation time (τ_2) first increases sharply and then reaches saturation at a certain high dose, which is explained by the limited number of deep energy defects formed under irradiation. The dependence of the relaxation time (τ_2) on the γ -radiation fluence becomes stronger as the temperature decreases (down to 77 K). The influence of charge carrier concentration fluctuations on the relaxation process is studied, and it is established that as the resistivity of the initial material decreases—that is, at higher concentrations—the amplitude of fluctuations increases, leading to an increase in the relaxation time.

Keywords: Film; Si<B,S>; Si<B,Rh>; γ -radiation; photoconductivity; relaxation time; temperature; concentrations

Kirish

Bugungi kunda elektronika, fotonika va quyosh energetikasi sohalarida yuqori samaradorlikka ega bo'lgan yarimo'tkazgichli materiallarga talab ortib bormoqda. Bunday materiallarni olishning samarali usullaridan biri kompensatsiyalangan yarimo'tkazgichlarni yaratish hisoblanadi. Kompensatsiyalash usuli yarimo'tkazgichda qarama-qarshi turdagi qo'shimcha primesalar kiritish yo'li bilan amalga oshiriladi. Bu esa materialning o'tkazuvchanlik turini, konsentratsiyasini va elektr hamda fotoelektrik xususiyatlarini boshqarish imkonini beradi.

Kremniy asosida kompensatsiyalash orqali olinadigan materiallar orasida bor (B) bilan legirlangan kremniy (Si) oltingugurt (S) yoki rodiiy (Rh) atomlari bilan kompensatsiyalash alohida o'rin tutadi [1]. Bunday kompensatsiyalangan Si<B,S> va Si<B,Rh> monokristallari yuqori darajada fotosezgirlikka ega bo'lib, ular quyosh energetikasi, fotodetektorlar va radiatsiya ta'siriga chidamli elektron qurilmalarda keng qo'llanishi mumkin [2]. Biroq, bunday materiallardagi elektr o'tkazuvchanlik xususiyatlari vaqt davomida o'zgarishi, ya'ni relaksatsiya jarayonlari kuzatilishi ma'lum. Relaksatsiya jarayonlarining tabiatini chuqur tushunish orqali materialning fotosezgirliги, elektr o'tkazuvchanligini va radiatsiya ta'siriga chidamliligini yaxshilash mumkin [3]. Shu nuqtai nazardan, Si<B,S> va Si<B,Rh> materiallarida elektr o'tkazuvchanlikning uzoq muddatli relaksatsiya jarayonlarini o'rganish muhim ilmiy-amaliy ahamiyatga ega hisoblanadi. Shu munosabat bilan mazkur ish doirasida Si<B,S> va Si<B,Rh> monokristallarida elektr o'tkazuvchanlikning relaksatsiya jarayonlari har xil konsentratsiyalardagi kompensatsiyalash sharoitida tadqiq qilingan.

Tadqiqot usullari

Tadqiqotlarda boshlang'ich material sifatida bor (B) bilan legirlangan p -turdagi kremniy (Si) monokristallari tanlab olindi. Boshlang'ich materialning solishtirma qarshiligi Si<B,S> namunalari uchun $1 \div 10$ Om·sm, Si<B,Rh> namunalari uchun esa $7 \div 10$

Om·sm ni tashkil etdi. Namunalarni kompensatsiyalash jarayoni $1250 \div 1290$ °C harorat oralig'ida, termodiffuziya usulida, 20 soat davomida amalga oshirildi. Natijada Si<B,S> namunalari kompensatsiya qiluvchi oltingugurt (S) atomlari konsentratsiyasi $N_S = (0,2 \div 2) \cdot 10^{16} \text{ sm}^{-3}$, Si<B,Rh> namunalari esa rodiiy (Rh) atomlari konsentratsiyasi $N_{Rh} \approx 5 \cdot 10^{15} \text{ sm}^{-3}$ darajasida shakllantirildi. Kompensatsiyalash natijasida har ikki turdagi namunalarda o'ziga xos qarshilikning keskin ortishi kuzatilib, $\rho = (8 \div 10) \cdot 10^4$ Om·sm ga teng bo'lgan yuqoriomli kremniy monokristallari olindi.

Natijalar va muhokama

Elektr o'tkazuvchanlikning relaksatsiya jarayonlari. Ma'lumki, fotoo'tkazuvchanlik relaksatsiyasi – yarimo'tkazgichli materiallarda fotog'alayonlantirish to'xtatilganidan keyingi fototokning pasayish jarayoni bo'lib, u materialning elektr-optik xususiyatlarini tavsiflovchi muhim parametr hisoblanadi [4]. Ushbu relaksatsiya jarayoni Si<B,S> va Si<B,Rh> monokristallari uchun ikki bosqichli eksponensial qonuniyat orqali aniq tasvirlanishi mumkin [5]:

$$I_{\Phi} = A_1 e^{-t/\tau_1} + A_2 e^{-t/\tau_2} \quad (1)$$

Bunda I_{Φ} – vaqtga bog'liq fototok, A_1 va A_2 – fototokning boshlang'ich amplitudalari, τ_1 va τ_2 esa relaksatsiya jarayonining vaqt bo'yicha xarakterli (xos) doimiylari hisoblanadi. [6] tadqiqot ishlarining mualliflari bu parametrlar kompensatsiya qiluvchi kirishmaning turi va konsentratsiyasiga kuchli bog'liq bo'lib, odatda $A_1 \gg A_2$ va $\tau_1 \ll \tau_2$ sharti bajarilishni ta'kidlaganlar. Bu esa relaksatsiya jarayonida tezkor va sekin bo'lgan ikki xil mexanizm mavjudligidan dalolat beradi.

1-jadvalda Si<B,S> va Si<B,Rh> monokristallari uchun o'tkazilgan eksperimentlar natijasida aniqlangan relaksatsiya parametrlarining xarakterli qiymatlari taqdim qilingan bo'lib, ular kompensatsiya qilingan materiallarning o'ziga xos xususiyatlarini aks ettiradi. Ushbu jadval ma'lumotlari materiallarning fotosezgirlik va

tezkorlik xususiyatlarini baholash hamda kompensatsiyalash orqali fotosezgirliги yuqori bo'lgan yarimo'tkazgichli materiallarni yaratish va ularni amaliy maqsadlarda qo'llash istiqbollarini ochib beradi [7].

1-jadval.

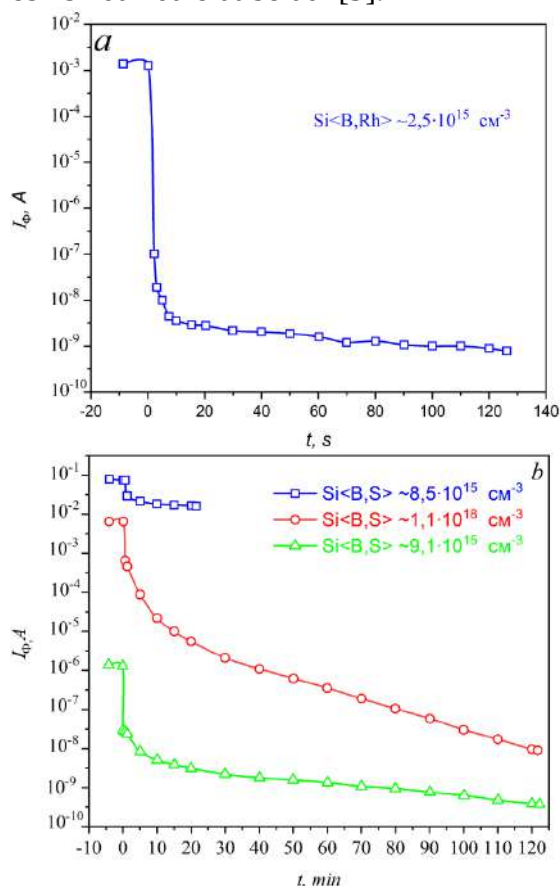
Fotoo'tkazuvchanlik relaksatsiyasi parametrlari

Namuna turi	Ifodasi	τ_1	τ_2
Si<B,S> (1)	$I_\phi = 4 \cdot 10^{-2} e^{\frac{t}{9}} + 2,5 \cdot 10^{-2} e^{\frac{t}{6184}}$	9	6184
Si<B,S> (2)	$I_\phi = 1 \cdot 10^{-7} e^{\frac{t}{261}} + 2 \cdot 10^{-8} e^{\frac{t}{4121}}$	261	4121
Si<B,S> (3)	$I_\phi = 1 \cdot 10^{-3} e^{\frac{t}{160}} + 5 \cdot 10^{-5} e^{\frac{t}{1060}}$	160	1060
Si<B,Rh>	$I_\phi = 2 \cdot 10^{-5} e^{\frac{t}{0,72}} + 4,5 \cdot 10^{-8} e^{\frac{t}{53}}$	0,72	53

Yuqoridagi jadvaldan ko'rinib turibdiki, Si<B,Rh> namunasida relaksatsiya vaqti Si<B,S> namunasiga nisbatan ancha qisqa bo'lib, bu materiallarda potensial to'siqning balandligiga bog'liqligi bilan izohlanadi.

Ma'lumki, fotoo'tkazuvchanlikning relaksatsiya jarayoniga tashqi omillarning, xususan, γ -nurlanish dozasi ta'sirini tahlil qilish muhim ilmiy ahamiyatga ega va bu materiallarning radiatsiya ta'siriga chidamliligini baholashda asosiy mezonlardan biri bo'lib xizmat qiladi [8]. Ushbu tadqiqot doirasida ham Si<B,S> va Si<B,Rh> namunalarda fototokning relaksatsiya jarayoniga turli dozadagi γ -kvantlarning ta'siri o'rganildi. Natijalar 1-rasmda grafik ko'rinishida tasvirlangan bo'lib, unda γ -kvantlar dozasi bilan fototok relaksatsiyasining ikkinchi xarakterli vaqti (τ_2) o'zgarishi aks ettirilgan. Olingan natijalarning tahlillari yordamida γ -kvant dozasi ortishi bilan relaksatsiyaning ikkinchi xarakterli vaqti (τ_2) dastlab sezilarli darajada oshib boradi, keyin esa ma'lum bir yuqori dozada to'yinish holatiga yetishi aniqlandi. Bunda to'yinish darajasi kompensatsiya qiluvchi atom turi (Rh yoki S) va ularning konsentratsiyasi bilan belgilanadi. Bu esa o'z navbatida kompensatsiyalangan Si<B,S> va Si<B,Rh> monokristallarida fotoqo'zg'atish ta'siri

ostida paydo bo'lgan tok tashuvchilarning qayta rekombinatsiyalanishi ikki xil vaqt xarakteriga ega jarayonlar orqali amalga oshirishidan dalolat beradi [9].



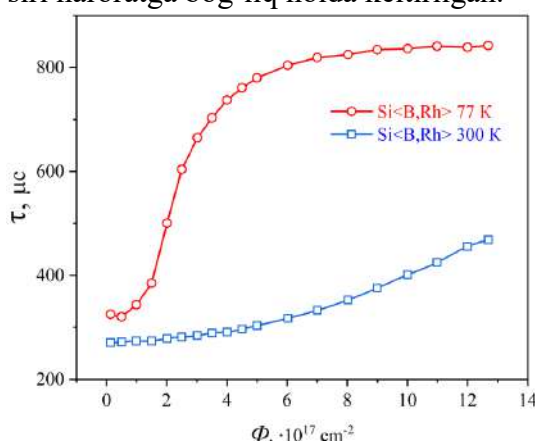
1-rasm. Fotoo'tkazuvchanlik relaksatsiya kinetikasining γ -kvantlar dozasi ta'siriga bog'liqligi. a) Si<B,Rh> namunasi $N_{Rh}^d \sim 2,5 \cdot 10^{15} \text{ cm}^{-3}$; b) Si<B,S> namunasi 1 – $\sim 8,5 \cdot 10^{15} \text{ cm}^{-3}$, 2 – $\sim 9,1 \cdot 10^{15} \text{ cm}^{-3}$, 3 – $\sim 1,1 \cdot 10^{18} \text{ cm}^{-3}$

Bunda ikkinchi xarakterli relaksatsiya vaqti (τ_2) fototokning sekin pasayishi bilan bog'liq bo'lib, u asosan yarimo'tkazgichda shakllangan chuqur energetik markazlar bilan aniqlanadi. Gamma-nurlanish dozasi oshirilganda Si<B,S> va Si<B,Rh> materiallarida turli chuqur energetik tuzoqlar (defekt markazlar) hosil bo'ladi yoki ularning konsentratsiyasi ortadi [10]. Bu esa hosil bo'lgan zaryad tok tashuvchilarning chuqur markazlarda ushlanib qolishiga olib keladi. Shu sababli tok tashuvchilarning qaytar rekombinatsiyalanishi sekinlashib, τ_2 vaqt xarakterining uzayishi kuzatiladi. Biroq, nurlanish dozasi ma'lum kritik darajaga yetishi bilan, defektlarning yaratilishi va ularning konsentratsiyasi to'yinish darajasiga yetadi [11]. Ya'ni chuqur energetik markazlar soni va

ularda ushlanishi mumkin bo'lgan nositellar miqdori cheklangan bo'ladi. Shu nuqtadan keyin dozani oshirish fototok relaksatsiya vaqtiga deyarli ta'sir ko'rsatmaydi, ya'ni τ_2 relaksatsiya vaqti ma'lum yuqori dozadan keyin o'zining maksimal qiymatiga yetib, o'zgarishdan to'xtaydi (to'yinish holati). Bu jarayonning kechishi kompensatsiyalovchi kirishmaning turi (Rh yoki S) va ularning konsentratsiyasiga bog'liq, chunki har bir kirishma atomi kremniy kristalli panjarasida o'ziga xos defekt holatlarini shakllantiradi va nositellarning rekombinatsiya jarayonini turlicha boshqaradi [12]. Bu natija kompensatsiya qilingan Si<B,S> va Si<B,Rh> namunalarining radiatsiya ta'siriga chidamliligini baholashda muhim ahamiyat kasb etadi hamda ularni radiatsiyaga chidamli fotosezgir materiallar yaratishda qo'llash mumkinligini tasdiqlaydi [13].

Nurlanish va haroratning relaksatsiya jarayoniga ta'siri tahlili.

Yarimo'tkazgichlarda fotoo'tkazuvchanlik relaksatsiyasi jarayonini chuqur tushunish uchun tashqi omillar, jumladan, nurlanish flyuyensi va haroratning ta'sirini hisobga olish muhim ahamiyat kasb etadi. γ -kvantlar flyuyensi ortishi bilan yarimo'tkazgichlarda defektlar soni ortib, bu esa zaryad tashuvchilarning rekombinatsiya jarayonlarini o'zgartiradi [14]. 2-rasmda Si<B,Rh> namunasida γ -kvantlar flyuyensining fototok relaksatsiya vaqtini o'zgartirishga bo'lgan ta'siri haroratga bog'liq holda keltirilgan.



2-rasm. Si<B,Rh> namunasida relaksatsiya vaqti τ_2 ning γ -kvantlar nurlanishga va haroratga bog'liqligi: 1 – 77 K; 2 – 300 K.

Rasmda keltirilgan natijalarning tahlillari shuni ko'rsatadiki, past haroratlarda (77 K) γ -kvantlar nurlanishining ortishi bilan relaksatsiya vaqtining o'zgarish sur'ati sezilarli

darajada yuqori bo'lgan. Bu esa o'z navbatida past haroratlarda termik aktivlashgan zaryad tashuvchilarning konsentratsiyasi kamayib, rekombinatsiya jarayoni asosan nurlanish natijasida yaratilgan chuqur defekt markazlari orqali amalga oshishidan dalolat beradi. Natijada, har bir defekt markazning ta'siri kuchayib, relaksatsiya jarayonining sekinlashuvi yanada aniqroq namoyon bo'ladi. Yuqori haroratlarda (300 K) esa termik aktivlashgan tok tashuvchilar soni ko'payadi, bu esa zaryad tashuvchilarning defektlar bilan o'zaro ta'sirini susaytiradi va relaksatsiya vaqtining o'zgarish sur'atini pasaytiradi [16]. Shu sababli, yuqori haroratda relaksatsiya vaqtining flyuyensga bog'liq o'zgarishi nisbatan kichik bo'ladi. Shunday qilib, harorat pasayganda relaksatsiya vaqtining nurlanish flyuyensiga sezgirligi ortishi yarimo'tkazgichlarda past harorat sharoitida ishlovchi radiatsiyaga chidamli fotoelektron qurilmalar yaratishda muhim hisoblanadi.

Konsentratsiya fluktuatsiyalarining relaksatsiya jarayoniga ta'siri.

Yarimo'tkazgich materiallarni kompensatsiyalash jarayonida zaryad tashuvchilarning konsentratsiyasida vujudga keladigan fluktuatsiyalar, ya'ni konsentratsiyaning o'zgaruvchanligi elektr o'tkazuvchanlik va fototok relaksatsiyasiga sezilarli ta'sir ko'rsatadi. Bu fluktuatsiyalar kompensatsiyalangan yarimo'tkazgichlarda energiya zonalarining lokal buzilishlari, ya'ni potensial baryerlarning shakllanishiga olib keladi.

2-jadval.

Kompensatsiya darajasi $K=1$ bo'lgan Si<B,S> namunalarida konsentratsiya fluktuatsiyalarining material turiga bog'liqligi

№	p^{max}, cm^{-3}	p^{min}, cm^{-3}	$\bar{p}, \text{cm}^{-3}$	$\frac{p^{max} - p^{min}}{\bar{p}} \cdot 100\%$	$\bar{N}_S, \text{cm}^{-3}$
1	$2,1 \cdot 10^{16}$	$1,9 \cdot 10^{16}$	$2 \cdot 10^{16}$	10	$2 \cdot 10^{16}$
2	$2,1 \cdot 10^{15}$	$1,9 \cdot 10^{15}$	$2 \cdot 10^{15}$	10	$2 \cdot 10^{15}$

Natijada zaryad tashuvchilarning harakatchanligi pasayadi va ularning rekombinatsiya jarayoni sekinlashadi, bu esa relaksatsiya vaqtining orta borishiga sabab bo'ladi. 2-jadvalda kompensatsiya darajasi $K=1$ bo'lgan Si<B,S> materiallari uchun zaryad

tashuvchilar konsentratsiyasi fluktuatsiyalarining amplitudasi va uning o'rtacha qiymatiga nisbatan o'zgarishi keltirilgan.

Jadval ma'lumotlaridan ko'rinadiki, zaryad tashuvchilarining konsentratsiya fluktuatsiyasi mutlaq qiymatda yuqori konsentratsiyalarda (ya'ni, boshlang'ich materialning udel qarshiligi past bo'lganda) katta bo'lib, bu esa potensial baryerlar balandligi va sonining orta borishiga olib keladi. Natijada zaryad tashuvchilarining rekombinatsiya vaqti uzayadi, shu bilan birga fotoo'tkazuvchanlik relaksatsiya vaqti ham oshadi. Shunday qilib, boshlang'ich material udel qarshiligining kamayishi zaryad tashuvchilarining konsentratsiya fluktuatsiyalarini kuchaytiradi va relaksatsiya vaqtining orttirilishi orqali materialning fotosezgirlik xususiyatlarini yaxshilash imkoniyatini yaratadi. Bu esa fotodetektor va yuqori sezgirlikka ega bo'lgan sensorlar ishlab chiqish uchun muhim hisoblanadi.

Xulosa

Olib borilgan tadqiqotlar va tahlillar asosida quyidagi asosiy ilmiy-amaliy xulosalarga kelindi:

fotoo'tkazuvchanlikning relaksatsiya jarayoni kompensatsiya qilingan Si<B,S> va Si<B,Rh> monokristallarida tezkor (τ_1) hamda sekin (τ_2) xarakterli vaqtlarga ega ikki bosqichli eksponensial qonuniyat bilan tavsiflanishi hamda bu relaksatsiya jarayonlari kompensatsiya qiluvchi kirishmaning turi va uning konsentratsiyasiga bog'liq ekani aniqlandi;

ryelaksatsiya parametrlari (τ_1 , τ_2) aniqlanib, Si<B,Rh> namunasida fototok relaksatsiyasining xarakterli vaqti Si<B,S> namunasiga nisbatan ancha qisqa ekanligi aniqlandi;

γ -nurlanish dozasi oshib borishi bilan relaksatsiyaning ikkinchi xarakterli vaqti (τ_2) dastlab keskin ortadi, keyin esa ma'lum bir yuqori dozada to'yinish holatiga yetishi aniqlandi va bu holat nurlanish ta'siri natijasida hosil bo'ladigan chuqur energetik defektlar sonining cheklanganligi bilan izohlandi;

ryelaksatsiya vaqti (τ_2)ning γ -nurlanish flyuyensiga bog'liq o'zgarishi harorat pasayganida (77 K) yanada kuchayishi

aniqlandi va bu holat past haroratlarda rekombinatsiya jarayoni asosan nurlanish ta'siri natijasida hosil bo'lgan defekt markazlari orqali amalga oshishi bilan izohlandi;

zaryad tashuvchilarning konsentratsiyasidagi fluktuatsiyalarning relaksatsiya jarayoniga ta'siri o'rganilib, boshlang'ich materialning solishtirma qarshiligi kamayganda, ya'ni yuqori konsentratsiyalarda fluktuatsiya amplitudasi oshib borishi va bu holatning relaksatsiya vaqtini uzaytirishi aniqlandi.

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Co-ION IMPLANTATSIYASINING ZnO VA Mg-BILAN LEGIRLANGAN ZnO YUPQA PLYONKALARIGA TA'SIRINI MONTE KARLO SRIM USULI YORDAMIDA MODELLASHTIRISH

H.Mansurov, N.Yunusaliyev, G'.Tojiboyev, N.Adixamov, M.Arabboyeva

Z. M. Bobur nomidagi Andijon davlat universiteti, O'zbekiston, 170100, Andijan sh, Universitet ko'chasi 129 uy

nyunusaliyev1997@gmail.com

2025-yil 17-avgustda berilgan; 2025-yil 1-noyabrda chop etish uchun qabul qilingan

Annotatsiya. 1,25 MeV energiyali Co ionlarining Si, ZnO va Mg bilan legirlangan ZnO (ZnO:Mg) namunaviy materiallari bilan o'zaro ta'siri SRIM 2013 dasturining so'nggi versiyasi yordamida tizimli ravishda tahlil qilindi. Tadqiqot natijalari shuni ko'rsatadiki, atomlarning siljishi, vakansiyalar hosil bo'lishi hamda energiya yo'qotilishi namunaviy materialining tarkibi va tuzilmaviy zichligiga kuchli darajada bog'liq. Olingan ma'lumotlarga ko'ra, eng yuqori nuqsonlar konsentratsiyasi kremniy kristallida kuzatiladi, buning sababi uning past ko'chirish energiyasi va kichik atom massasi bo'lib, ular qaytish (recoil) to'qnashuvlari ehtimolini oshiradi. Aksincha, ZnO yuqori atom zichligi va mustahkam bog'lanish energiyasi tufayli panjara tartibsizlanishiga nisbatan yaxshiroq bardoshlilik namoyon qilib, o'rtacha darajada nuqsonlar generatsiyasini ko'rsatadi. Undan tashqari, ZnO matritsasiga Mg kiritilishi umumiy zarar miqdorini biroz kamaytiradi, bu esa panjaraning nisbatan barqaror va radiatsiyaga chidamli bo'lishini anglatadi. Ushbu natijalardan kelib chiqadiki, Mg bilan legirlash ZnO strukturaviy mustahkamligini yuqori energiyali Co-ion bombardimoniga nisbatan sezilarli darajada oshiradi va natijada ZnO:Mg plyonkalarini radiatsiyaga chidamli optoelektronika hamda sensor qurilmalari uchun yanada maqsadga muvofiq qiladi.

Kalit so'zlar: To'xtatish qobiliyati, ZnO, Mg bilan legirlangan ZnO, SRIM, ion implantatsiyasi, vakansiya, nuqson, radiatsion shikastlanish, Monte Karlo simulatsiyasi.

Аннотая. Поведение взаимодействия ионов Co с энергией 1,25 МэВ с мишенями Si, ZnO и ZnO:Mg было систематически проанализировано с использованием последних симуляций SRIM 2013. Полученные результаты показывают, что атомные смещения, образование вакансий и потери энергии в значительной степени зависят от состава и структурной плотности мишени. Установлено, что наибольшая концентрация дефектов наблюдается в кристаллическом кремнии (Si) из-за его низкой пороговой энергии смещения и меньшей атомной массы, что повышает вероятность столкновений отдачи. В то же время в ZnO наблюдается умеренное образование дефектов, поскольку высокая атомная плотность и энергия связи усложняют разупорядочение решётки. Кроме того, присутствие магния (Mg) в матрице ZnO слегка снижает общее повреждение, что указывает на большую стабильность и радиационную стойкость решётки. Из этих результатов можно сделать вывод, что легирование Mg действительно повышает структурную прочность ZnO при облучении высокоэнергетическими ионами Co, делая плёнки ZnO:Mg более подходящими для радиационно-стойких оптоэлектронных и сенсорных применений.

Ключевые слова: Тормозная способность, ZnO, ZnO легированный магнием, SRIM, ионная имплантация, вакансия, дефект, радиационное повреждение, моделирование Монте-Карло.

Abstract. The interaction behavior of 1.25 MeV Co ions with Si, ZnO, and Mg-doped ZnO (ZnO:Mg) targets has been systematically analyzed using the latest SRIM 2013 simulations. The findings show that atomic displacements, the production of vacancies and energy loss are greatly influenced by the composition and structural density of the target. The results reveal that the highest defect concentration is seen in crystalline Si due to its lower displacement threshold energy and smaller atomic mass which increases the likelihood of recoil collisions. On the other hand, ZnO shows moderate defect generation as its high atomic density and bonding energy make it harder for the lattice to be disordered. Moreover, the presence of Mg in the ZnO matrix reduces the overall damage slightly, meaning the lattice is more stable and tolerant to radiation. It can be inferred from these results that Mg doping does indeed improve the structural robustness of ZnO against high-energy Co-ion bombardment and thus makes ZnO:Mg films more appropriate for radiation-resistant optoelectronic and sensing applications.

Keywords: Stopping power, ZnO, Mg-doped ZnO, SRIM, ion implantation, vacancy, defect, radiation damage, Monte Carlo simulation.

I. KIRISH. So'nggi yillarda ion implantatsiyasi usullari yarimo'tkazgich materiallarining fizik, strukturaviy va elektron xossalarini boshqarishda eng keng qo'llaniladigan texnologiyalardan biriga

aylangan. Ionlarning energiyasi, oqimi va turini aniq nazorat qilish imkoniyati material xususiyatlarini atom darajasida o'zgartirishga sharoit yaratadi. Bu, ayniqsa, yuqori radiatsiya darajasida yoki ekstremal

sharoitlarda ishlaydigan materiallarni yaratish uchun alohida ahamiyatga ega. Ushbu maqsadlar uchun o'rganilgan turli ionlar orasida kobalt (Co) o'rta atom massasi, elektron va yadro tizimlari bilan kuchli o'zaro ta'sirlashishi, shuningdek, oksid asosidagi yarimo'tkazgichlarda bir vaqtning o'zida ham magnit, ham strukturaviy o'zgarishlar keltirib chiqarish qobiliyati bilan ajralib turadi [1]. Sink oksidi (ZnO) — II–VI guruhiga mansub, keng taqiqlangan energiya sohasiga ega ($E_g \approx 3,3$ eV) birikma hisoblanadi. Yuqori optik shaffoflik, pyezoelektr xususiyatlar va kimyoviy barqarorlik kabi o'ziga xos xususiyatlari tufayli u optoelektron qurilmalar [2], gaz sensorlari [3], ultrabinafsha fotodetektorlari [4] hamda shaffof o'tkazgich parda texnologiyalarida [5] asosiy material sifatida keng qo'llaniladi. Biroq, ZnO yuqori energiyali zarrachalar yoki nurlanish ta'siriga tushganda kristall tuzilmasi o'zgarishi, turli nuqsonlar hosil bo'lishi mumkin [5]. Bunday nurlanish natijasida yuzaga keladigan nuqsonlar — masalan, kislorod vakansiyalari, ruxning interstitsial atomlari yoki murakkab nuqsonlar — uning elektr o'tkazuvchanligi, optik xossalari, tashuvchilar harakatchanligi va hatto o'tkazuvchanlik turini sezilarli darajada o'zgartirishi mumkin [6]. Shu sababli, ZnO materiallarining radiatsiyaga chidamliligini oshirish va nuqsonlarning barqarorligini ta'minlash zamonaviy yarimo'tkazgichlar fizikasi oldidagi eng dolzarb masalalardan biri hisoblanadi. ZnOning radiatsiyaga chidamliligini oshirishning eng samarali usullaridan biri — uning katyon panjarasiga boshqa elementlarni kiritish (legirlash) bo'lib, bu jarayonda ayniqsa magniy (Mg) atomlari keng qo'llaniladi. Mg^{2+} kationlarining Zn^{2+} ostpanjarasiga qo'shilishi taqiqlangan zona kengligining sezilarli oshishiga olib keladi [7], bu esa kristallning qattiqligini yaxshilashi va ZnO matritsasining umumiy panjara barqarorligini oshirishi bilan birga kechadi [8]. Shu sababli Mg bilan legirlangan ZnO (ZnO:Mg) radiatsiya va yuqori harorat sharoitida barqaror ishlaydigan optoelektron va sensor tizimlari uchun istiqbolli material sifatida qaraladi. Shunga qaramay, yuqori energiyali ionlarning sof va legirlangan ZnO tizimlari bilan o'zaro ta'sir jarayoni yetarlicha o'rganilmagan. Bu jarayonni chuqur tushunish uchun ionlar va qattiq jism atomlari o'rtasidagi o'zaro ta'sir mexanizmlarini atom darajasida tahlil qilish talab etiladi. 1,25 MeV energiyaga ega Co ionlari qattiq jism orqali o'tganda, ular energiyasini asosan ikki asosiy jarayon orqali yo'qotadi: elektronik to'xtatish (atomlarni elektronlari bilan noelastik to'qnashuvlar) va yadroviya to'xtatish (atom yadro-lari bilan elastik to'qnashuvlar) [9]. Ushbu ikki jarayonning nisbati ionning kirib borish

chuqurligi (ion diapazoni) hamda hosil bo'ladigan strukturaviy shikastlanishning xarakteri va fazoviy taqsimotini belgilaydi. Energiya sochilish darajasi va atomlarning qaytish jarayonlari ionning massasi va zaryadi bilan birga materialni zichligi, atom massasi va bog'lanish energiyasiga ham bog'liq. Ana shu parametrlar materialning radiatsiya ta'sirida qanday o'zgarishini va uning ishlash barqarorligini belgilaydi. Ushbu o'zaro ta'sir jarayonlarini aniq tasvirlash va prognozlash uchun Monte Karlo turidagi hisoblash modellari, xususan, SRIM (Stopping and Range of Ions in Matter) dasturiy ilovasi keng qo'llaniladi [10]. SRIM simulyatsiyalari ion trayektoriyalari, to'xtatish qobiliyatlari, qaytish (recoil) taqsimoti, ion diapazoni va nuqsonlar hosil bo'lish tezligi kabi parametrlarga aniq baho berish imkonini yaratadi. Bu energiya taqsimlanishi jarayonlari va panjara buzilish mexanizmlarini mikroskopik miqyosda tahlil qilishga imkon beradi. Eksperimental o'lchovlar ko'pincha murakkab va cheklangan sharoitlarda bajarilsa, SRIM asosidagi hisob-kitoblar turli material tizimlarini taqqoslash uchun to'liq nazariy, ammo juda informativ yondashuvni taqdim etadi. Ushbu ishda 1,25 MeV energiyaga ega Co ionlarining bir kristalli kremniy (Si), sof ZnO va Mg bilan legirlangan ZnO (ZnO:Mg) yupqa plyonkalar bilan o'zaro ta'siri Monte Karlo SRIM-2013 simulyatsiyalari yordamida modellash tirildi. Ushbu materiallarning tanlanishi turli atom massasi va tuzilmaviy xususiyatlarga ega yarimo'tkazgichlarning radiatsiya natijasida reaksiyasini qiyosiy tahlil qilish imkonini beradi. Tadqiqot asosan energiya yo'qotish mexanizmlarini, to'xtatish qobiliyatining (stopping power) xatti-harakatini va shikastlanish hosil bo'lish tendensiyalarini baholashga qaratilgan. Ayniqsa, ZnO panjarasiga Mg kiritilishi natijasida kobalt ionlari bilan radiatsiyalash jarayonida energiya tarqalishi yo'llari va strukturaviy mexanizmlarida qanday o'zgarishlar yuz berishi batafsil o'rganildi. Olingan natijalar oksid yarimo'tkazgichlarning radiatsiyaga chidamli variantlarini ishlab chiqish uchun muhim nazariy asos bo'lib xizmat qiladi.

II. MATERIAL VA USULLAR

Simulyatsiya vositalari ionlarning qattiq jismlar bilan o'zaro ta'sirini tushunishda zamonaviy materialshunoslik va yarimo'tkazgichlar fizikasi uchun ajralmas qism hisoblanadi. Bunday hisoblash yondashuvlari orqali laboratoriya sharoitida aniqlash qiyin bo'lgan atom darajasidagi jarayonlarni hech bo'lmaganda qisman modellash tirish imkoniyati yaratiladi. Monte Karlo usuliga asoslangan eng mashhur va ishonchli dasturlardan biri — SRIM (Stopping and Range of Ions in Matter) hisoblanadi. SRIM dasturlari

yordamida ionlarning trayektoriyalari, energiya yo'qotish mexanizmlari va murakkab ko'p qatlamli strukturalarda nuqsonlar hosil bo'lishi aniqlanishi mumkin [11]. SRIM kodi Co ionlarining Si, ZnO va ZnO:Mg yupqa plyonkalari orqali o'tish jarayonida energiya yo'qotishi va sochilishini baholash uchun yagona, o'zaro uyg'unlashgan nazariy asos yaratadi. Ushbu tadqiqotda SRIM-2013 ilovasi yordamida 1,25 MeV energiyali Co ionlarining yuqorida tilga olingan uch xil materialga implantatsiyasi modellashtirildi va ular uchun to'xtatish qobiliyati, ion diapazoni hamda strukturaviy shikastlanish xarakteri tahlil qilindi. Dastur ionlarning har birining yo'lini tasodifiy elastik va noelastik to'qnashuvlar ketma-ketligi sifatida simulyatsiya qiladi. Har bir to'qnashuvda ionning kinetik energiyasi kamayadi va bu energiya ikki jarayon o'rtasida taqsimlanadi: yadroviy to'xtatish (elastik sochilish) hamda elektronik to'xtatish (ion–elektron o'zaro ta'siri orqali noelastik sochilish). Ionlar material ichidan o'tayotganda asta-sekin energiyasini yo'qotadi, shu bilan birga mayda burchakli sochilishlar tufayli ularning yo'nalishida yengil og'ishlar ham sodir bo'ladi. Bir birlik masofa uchun umumiy energiya yo'qotilishi (to'xtatish qobiliyati, $-dE/dx$) materialning ion energiyasini qanchalik yaxshi so'ra olishi yoki tarqatishini belgilovchi muhim parametrdir [12,13]. Yuqori energiyalarda, bu holatda bo'lgani kabi, elektron to'xtatish doimo ustun jarayon hisoblanadi, chunki Co ionlari energiyasini asosan ionizatsiya va qo'zg'atish orqali yo'qotadi. Aynan shu jarayon Co ionlarining Si va ZnO panjaralaridagi yo'llarini belgilab beradi. Ion energiyasi kamaygan sari, ayniqsa diapazonning oxirgi qismiga (EOR) yaqinlashganida, yadroviy to'xtatish roli ortib boradi, bu esa katta atom siljishlari va vakansiyalar hosil bo'lishiga olib keladi [14]. Simulyatsiyalar materiallarning nurlanish vaqtida va undan keyingi holatdagi reaksiyalarini oldindan baholashda juda muhimdir, ayniqsa ion implantatsiyasi dopant kiritish va interfeyslarni modifikatsiya qilish uchun keng qo'llaniladigan yarimo'tkazgich texnologiyalarida. SRIMning to'xtatish qobiliyatlari, qaytish (recoil) taqsimotlari va ionizatsiya profillarini aniqlik bilan hisoblay olishi mikroelektronika, yupqa plyonka texnologiyasi va radiatsion himoya tadqiqotlari uchun uni ajralmas vositaga aylantiradi. SRIMdan tashqari TRIM, TRIDYN va SDTrimSP kabi boshqa Monte Karlo paketlari ham ion–qattiq jism o'zaro ta'sirini modellashtirishda qo'llaniladi; biroq SRIM o'zining yaxshi tasdiqlangan fizik modellariga va qulay interfeysiga ko'ra eng ko'p ishlatiladigan dastur bo'lib qolmoqda. Nazariy jihatdan SRIM simulyatsiyalari tez zaryadlangan zarrachalarning qattiq jismlardagi elektron energiya yo'qotilishini

ifodalovchi Bethe–Bloch tenglamasiga ham mos keladi. Bethe–Bloch doimiy sekinlashuvni ifodalasa, SRIM undan tashqari atomlarning qaytishlari, burchakli sochilishlar va nuqsonlar kaskadlari kabi stoxastik jarayonlarni ham hisobga oladi. Bu yondashuv ionlarning keltirib chiqaradigan shikastlanishlarni fazoviy va energetik taqsimoti bo'yicha to'liq tasavvur hosil qiladi. Ushbu tadqiqot holatida SRIM simulyatsiyalari materiallarning atom zichligi va strukturaviy qattiqligi bilan mos ravishda $Si \rightarrow ZnO \rightarrow ZnO:Mg$ ketma-ketligida umumiy zarar energiyasi, vakansiyalar miqdori va atomlar siljishining kamayishini ko'rsatdi. Dastlabki bosqichlarda elektron to'xtatish, diapazon oxiriga yaqin esa yadroviy to'qnashuvlarining kuchayishi har bir material uchun xos bo'lgan Braggga o'xshash energiya yutilish profillarini yuzaga keltiradi. Bu hodisalarni chuqur tushunish radiatsiyaga chidamli yarimo'tkazgichlarni yaratish va material xossalarni tarkibiy modifikatsiya orqali boshqarish uchun juda muhimdir. Shu bois SRIM asosidagi Monte Karlo yondashuvi Co ionlari ta'siridagi energiya yo'qotilishini vizuallashtirish va miqdoriy baholashda kuchli nazariy-amaliy platforma hisoblanadi. Bunday hisoblash natijalari nafaqat radiatsiya–modda o'zaro ta'sirini tushuntirish, balki nuqsonlar arxitekturasini nazorat qilish imkoniga ega samarali oksid yarimo'tkazgichlarni ishlab chiqarishda ham katta ahamiyatga ega. Shu ma'noda, SRIM bugungi zamonaviy materialshunoslikda nazariy modellar, eksperimental tasdiqlar va texnologik qo'llanmalarining o'zaro bog'lanishini ta'minlab beruvchi muhim vosita hisoblanadi.

III. ZnO va ZnO:S Yupqa Plyonkalarini Tayyorlash Jarayoni

1,25 MeV energiyali kobalt (Co) ionlarining implantatsiya simulyatsiyasi SRIM 2013 dasturiy paketi yordamida uch xil nishon materiali — kremniy (Si), sink oksidi (ZnO) va magniy bilan legirlangan sink oksidi (ZnO:Mg) uchun bajarildi. Tadqiqotda qo'llanilgan fizik parametrlar eksperimental ravishda e'lon qilingan qiymatlar hamda tasdiqlangan adabiyot ma'lumotlariga asoslangan holda tanlandi [6,15]. Simulyatsiya geometriyasida bir xil zichlikka ega Co ionlar nuri tekis holatdagi nishon qatlamlariga perpendikulyar yo'nalishda yo'naltirilgan bo'lib, bu 1-rasmda tasvirlangan. Kristall kremniyning zichligi 2,321 g/sm³ qilib belgilandi, sof ZnO ning zichligi esa 4,074 g/sm³ etib o'rnatildi. Mg bilan legirlangan ZnO qatlamining zichligi 3,972 g/sm³ gacha kamaytirildi, bu esa Zn (44,3 %), O (51,7 %) va Mg (4,0 %) atom ulushlariga mos keladi. Elementar tarkibga asoslangan holda siljitish energiyalari

quyidagicha belgilandi: Si uchun 15 eV, ZnO asosidagi tizimlar uchun esa rux (Zn) atomlari uchun 25 eV va kislorod (O) atomlari uchun 28 eV. Shuningdek, Zn–O va Zn–Mg panjaralarining kuchli ionik-kovalent bog‘lanish tabiatini hisobga olgan holda sirt va panjara bog‘lanish energiyalari ham mos ravishda o‘zgartirildi.

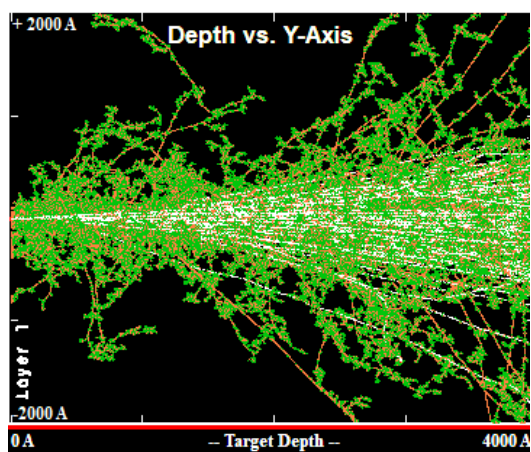
1-jadval. Namunalar kimyoviy formulasi.

Materiallar	Kimyoviy Formulalar
Kremniy substrati	Si
Rux oksidi	ZnO
Magniy bilan legirlangan rux oksidi	ZnO:Mg

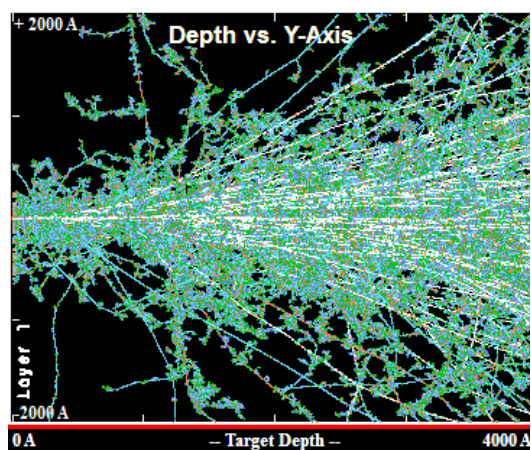
Turli nishon qatlamlari va qaytgan atomlarning holati ushbu grafiklar orqali (1-rasm) tasvirlangan. Chizilgan trayektoriyalar ionlarning modda ichiga kirib borishi jarayonida sodir bo‘lgan energiya sarfi, sochilish va materialdagi o‘zgarishlarni aks ettiradi. Kremniy (1a-rasm) Co ionlariga eng uzun va eng keng tarqalish imkonini berdi — ionlar substratga chuqur kirib bordi va yonlama siljishlar (lateral straggle) ham juda katta bo‘ldi. Bu xatti-harakat kremniyning nisbatan kichik atom massasi va past zichligi ($2,321 \text{ g/sm}^3$) bilan izohlanadi. Bunday sharoit tushayotgan ionlarga kamroq qarshilik ko‘rsatadi. Shu sababli trayektoriya maydoni yanada qalin va cho‘zilgan ko‘rinishga ega bo‘lib, ionlar sezilarli sekinlashuv sodir bo‘lishidan avval ham katta qism kinetik energiyasini saqlab qolayotganini ko‘rsatadi. Katta sochilish burchaklari taqsimoti shuni ko‘rsatadiki, diapazonning oxirgi qismiga yaqin elastik yadro to‘qnashuvlari energiya sochilishiga sezilarli darajada hissa qo‘shadi. Bu esa kremniy matritsasi ichida nuqsonlar konsentratsiyasining ortishi va atom siljishlari zichligining yuqori bo‘lishiga olib keladi. SRIM hisob-kitoblari ko‘ra, Co ionlarining kremniydagi o‘rtacha proyeksiyalangan diapazoni taxminan 1,8 mkm ga yetgan, bu esa o‘rganilgan materiallar ichida Si eng chuqur kirib borish yo‘liga va eng katta tarqalishga (longitudinal stragglings) ega ekanligini tasdiqlaydi. Ionlarning bunday uzoq masofaga kirib borishi Si ning past to‘xtatishi bilan bevosita bog‘liq bo‘lib, Co ionlari sezilarli sekinlashuv boshlanishidan oldin ham ancha masofagacha yuqori kinetik energiyani saqlab qoladi. Natijada, kremniy ichida eng cho‘zilgan to‘qnashuvlar va eng katta shikastlanish sohasi shakllanadi. Aksincha, ZnO uchun (1b-rasm) ion trayektoriyalari Si dagiga nisbatan kuchsizroq, qisqaroq va ixchamroq shakllangan. ZnO ning atomlari zich joylashgan ($4,074 \text{ g/sm}^3$) va Zn–O orasidagi ion-kovalent bog‘lanish kuchli bo‘lgani uchun ionlarning kirib borish chuqurligi ham, sochilish amplitudasi ham sezilarli darajada sekinlashadi. SRIM natijalariga

ko‘ra, Co ionlarining katta qismi taxminan 1,2 mkm (yoki yupqa plyonka geometriyasida $\approx 276 \text{ nm}$) proyeksiyalangan diapazonda to‘xtaydi, bu jarayonda energiya yo‘qotilishi asosan elektron to‘xtatishdan yadroviya to‘xtatishga o‘tish bosqichida sodir bo‘ladi. Zn va O atomlarining yuqori atom massalari qaytish (recoil) harakatini cheklaydi, shuning uchun to‘qnashuvlar juda kichik hajmda saqlanib qoladi. Natijada ZnO strukturasi atomlar siljishi o‘rtacha darajada bo‘ladi va markaziy to‘qnashuv zonasi yaqqol ajralib turadi, undan so‘ng energiyaning tez so‘nishi kuzatiladi. Ushbu holat ZnO ning kuchli ion-kovalent bog‘lanishi kremniyga nisbatan struktura tartibsizlanishiga ko‘proq qarshilik ko‘rsatishini tasdiqlaydi. Bu esa kamroq nuqson zichligi va sayozroq ion yo‘li bilan ifodalanadi. Bundan tashqari, Zn uchun 25 eV va O uchun 28 eV bo‘lgan o‘rtacha siljitish energiyalari ko‘pchilik qaytish hodisalari ion diapazoni oxiri (EOR) mintaqasida lokalizatsiyalanishini ta‘minlaydi. ZnO:Mg tizimida (1c-rasm) Co ionlarining yo‘llari yanada bir tekis tarqalgan bo‘lib, diapazon oxiri (EOR) mintaqasida ularning zichligi ancha past kuzatiladi. Mg atomlarining mavjudligi panjara tuzilmasiga ta‘sir ko‘rsatib, o‘rtacha elektron zichligini biroz kamaytiradi. Natijada elektronik va yadroviya to‘xtatish ehtimollari ham pasayadi. Shuning uchun hosil bo‘lgan sochilish yanada silliq va bir xil xarakterga ega bo‘ladi. Legirlash natijasida hosil bo‘lgan plyonkada tarqalgan to‘qnashuv zanjirlari soni kamroq va yonlama tarqalish (lateral straggle) darajasi past bo‘lib, bu materialning yuqori energiyali ion zarbasiga nisbatan ko‘proq strukturaviy barqarorlik ko‘rsatishini anglatadi. SRIM hisob-kitoblari ZnO:Mg uchun o‘rtacha proyeksiyalangan diapazonni 1,0–1,1 mkm deb baholadi, bu o‘rganilgan barcha materiallar ichida eng sayoz kirib borish chuqurligi ekanini ko‘rsatadi. Ion yo‘lining qisqarishi effektiv to‘xtatish qobiliyati oshishi hamda panjara barqarorligining kuchayishi bilan bevosita bog‘liq. Magniy atomlarining atom massasi kichik (24 amu) bo‘lgani sababli ular lokal energiya tamponlari sifatida harakat qiladi, qaytishlar kamaydi va nuqsonlar to‘planishini kamaytiradi. Natijada radiatsiyaga chidamlilik oshadi. Sof ZnO ga nisbatan ZnO:Mg recoil jarayonlari ta‘siriga kamroq uchraydi, energiya tarqalish profili esa kichikroq bo‘ladi. Bu esa uning panjarasi energiyani samaraliroq yutishi, o‘zini tiklash qobiliyati yaxshiroq bo‘lishi va kamroq shikastlanish to‘planishini bildiradi. Umuman olganda, ushbu ishda taqdim etilgan trayektoriya xaritalari ionlarning kirib borish chuqurligi va sochilish intensivligi quyidagi tartibda bo‘lishini aniq ko‘rsatadi: $\text{Si} > \text{ZnO} > \text{ZnO:Mg}$. Kremniy (Si) nafaqat eng ko‘p shikastlanishni boshdan kechiradi,

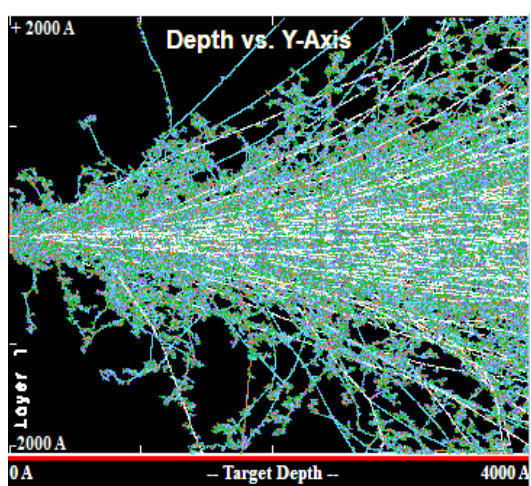
balki eng ekstremal darajadagi tartibsizlik va eng uzun ion diapazoniga ega. Aksincha, ZnO panjarasiga magniy kiritilishi (Mg legirlash) strukturaning mustahkamligini oshiradi va radiatsiyaga sezgirligini kamaytiradi.



a)



b)

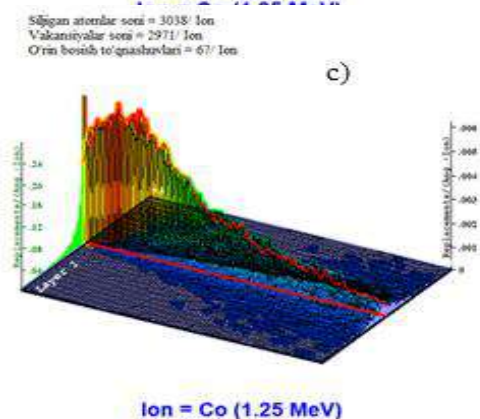
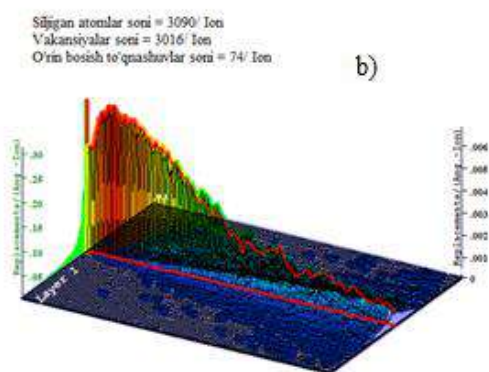
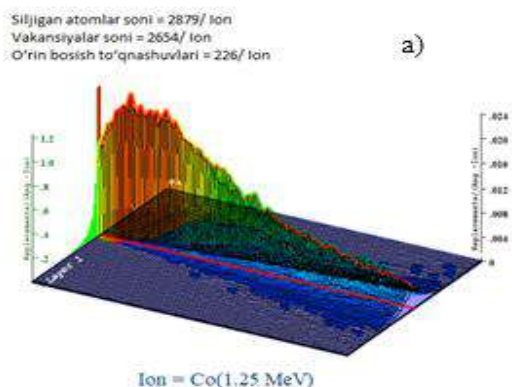


c)

1-rasm. SRIM tomonidan modellashtirilgan 1,25 MeV energiyali Co ionlarining (a) ZnO, (b) ZnO:S va (c) ZnO:S/Si moddalar trayektoriyalari. Rasmda ionlarning chuqurlik bo'ylab uzunlamasına taqsimoti va sochilishlar aks ettirilgan.

Bu natijalar moddiy tarkibni o'zgartirish — xususan Mg qo'shish — radiatsiya tufayli yuzaga keladigan nuqsonlar sonini kamaytirishda samarali ekanini yana bir bor tasdiqlaydi. Shu sababli ZnO:Mg, ion nurlanishi mavjud bo'lgan yarimo'tkazgich va yupqa plyonka texnologiyalarida yanada ishonchli material sifatida qaraladi. SRIM dasturi 1,25 MeV energiyali Co ionlari ta'sirida Si, ZnO va ZnO:Mg moddalarida hosil bo'ladigan umumiy atom siljishlari, vakansiyalar va almashtirish to'qnashuvlari (replacement collisions) bo'yicha hisob-kitoblarni taqdim etadi. Ushbu simulyatsiya ma'lumotlari har bir materialning nuqson hosil bo'lish tendensiyalarini qiyosiy tarzda yoritadi. Displacement (siljish) va vacancy (vakansiya) egri chiziqlari Co ionlarining yuqori energiyali sharoitda har bir materialning atom tartibsizlanishiga qarshilik darajasini ko'rsatadi (2-rasm). Kremniy (Si) holatida umumiy siljishlar soni eng yuqori qiymatga (taxminan 2879 ta har bir ionga) yetadi, bu esa Si eng oson shikastlanadigan material ekanini anglatadi. Ushbu juda katta nuqson hosil bo'lishi kremniyning past atom massasi va nisbatan zaif bog'lanish kuchlari bilan bog'liq bo'lib, ular Co ionlaridan panjara atomlariga kinetik energiyani oson o'tishini ta'minlaydi. Natijada, yadro to'qnashuvlari va elektron energiya yo'qotish jarayonlari vakansiyalar va interstitsial nuqsonlarning hosil bo'lishini sezilarli darajada kuchaytiradi. Yuqori vakansiya soni (~2654/ion) va katta o'rin bosish to'qnashuvlari soni (226/ion) kristall panjarada atomlar aralashuvi va kaskadlarning bir-biri bilan ustma-ust kelishi intensiv ravishda sodir bo'layotganini ko'rsatadi. Sink oksidi (ZnO) uchun umumiy atom siljishlari soni biroz pastroq (~3090/ion), biroq o'rin bosish to'qnashuvlari keskin kamayib, atigi 74/ionni tashkil etadi. Bu shuni anglatadiki, ZnO metall ionlar simulyatsiya natijasida Si ga nisbatan kamroq shikastlansa-da, uning yuqori zichligi va katta atom massalari (Zn = 65, O = 16) tufayli ion energiyasining anchayin qismini yutadi, ammo uning qattiq ion-kovalent bog'lari atomlarning migratsiyasiga yo'l qo'ymaydi. Shunday qilib, ZnOning samarali to'xtatish qobiliyati nafaqat atom siljishlarini cheklaydi, balki qaytish (recoil) ham sirtga yaqin ixcham hududlarda saqlab qoladi va katta miqyosli strukturaviy o'zgarishlarning oldini oladi. Bu ZnO yuqori energiyali Co ionlari ta'sirida o'rtacha darajadagi strukturaviy shikastlanish va kuchaytirilgan nuqson lokalizatsiyasini namoyon etishini tasdiqlaydi. Zn va O panjaralari nafaqat keng ko'lamli atom qayta tartiblanishlarini, balki chuqur to'qnashuvlarning shakllanishini ham cheklaydi. Shu sababli qaytish aksariyati ion diapazonining oxirgi (end-of-range) qismida sodir bo'ladi va bu chuqur emas, balki sirtga yaqin ixcham

shikastlanish zonalarini hosil qiladi. Mg bilan legirlangan ZnO (ZnO:Mg) da SRIM hisob-kitoblariga ko'ra sof ZnO ga nisbatan umumiy atom siljishlari soni yanada kamayadi va taxminan 2.56×10^3 siljish/ion qiymatiga teng bo'ladi. Ushbu kamayish Mg atomlarining ZnO panjarasi ichida nuqson kaskadlari shakllanishini samarali ravishda bostirishini va panjara shikastlanishini sezilarli darajada kamaytirishini ko'rsatadi. Yengil Mg atomlari "energiya tamponi" vazifasini bajarib, kirayotgan energiyani yumshatadi va shu bilan birga nuqsonlar to'planishini cheklaydi. Shunga qaramay,



vakansiya konsentratsiyasi ($\sim 2971/\text{ion}$) sof ZnO va Si ga nisbatan past bo'lib qolmoqda, bu esa ZnO:Mg ning nurlanishga nisbatan yuqori chidamliligini ko'rsatadi.

2-rasm. SRIM tomonidan modellashirilgan 1,25 MeV energiyali Co ionlari implantatsiyasi natijasida (a) Si, (b) ZnO va (c) ZnO:Mg materiallarda hosil bo'lgan umumiy siljishlar, vakansiyalar va o'rin bosish to'qnashuvlari.

Uchala material o'rtasidagi taqqoslash bir muhim faktni aniq ko'rsatadi: ionlar keltirib chiqaradigan tartibsizlik (disorder) samaradorligi $\text{Si} > \text{ZnO} > \text{ZnO:Mg}$ ketma-ketligida izchil ravishda kamayib boradi. Bu kamayish materiallarning atom bog'lanishi, zichligi va siljitish energiyasi kabi uchta asosiy omil bilan bevosita bog'liq. Kremniy manfiy zaryadlangan va kovalent bog'lanishga ega bo'lgani sababli qaytish (recoil) natijasidagi siljishlarga nisbatan ancha sezgir, aksincha, ion-kovalent xususiyatga ega bo'lgan Zn-O panjarasi bunday siljishlarga ko'proq chidamli. Eng qiziqarli jihat shundaki, ZnO ga Mg kiritilishi panjaraning umumiy tarqalish (scattering) kesimini oshirib, panjaradan atomlarning chiqib ketish harakatini sekinlashtiradi va shu bilan barqarorlikni yanada kuchaytiradi. Xulosa qilib aytganda, SRIM simulyatsiyalari Co ionlarining Si da eng yuqori darajadagi panjara nuqsonlarini, ZnO da o'rtacha darajadagi shikastlanishni, ZnO:Mg da esa eng past nuqson zichligini hosil qilishini ko'rsatdi. Mg bilan legirlangan ZnO uchun umumiy siljishlar va o'rin bosish to'qnashuvlari sonining kamayishi ushbu materialning ionlashtiruvchi nurlanish sharoitida ishlashi kerak bo'lgan ilg'or optoelektron va sensor qurilmalar uchun istiqbolli, radiatsiyaga chidamli oksid yarimo'tkazgich sifatidagi salohiyatini tasdiqlaydi.

IV.XULOSA.

Monte Karlo usuliga asoslangan SRIM-2013 simulyatsiyasi 1,25 MeV energiyali Co ionlarining Si, ZnO va ZnO:Mg nishonlarida to'xtatilish xatti-harakati, energiya tarqalishi va nuqsonlar hosil bo'lish jarayonlarini tahlil qilish uchun qo'llanildi. Hisoblash natijalari asosida quyidagi asosiy xulosalar chiqarildi. Ionlarning kirib borish chuqurligi material tarkibiga kuchli bog'liqlik ko'rsatdi. O'rtacha proyeksiya diapazoni qiymatlari Si uchun taxminan $\approx 1,8$ mkm, ZnO uchun $\approx 1,2$ mkm, hamda ZnO:Mg uchun $\approx 1,0-1,1$ mkm ni tashkil etdi. Umumiy atom siljishlari va vakansiyalar soni Si panjarasida eng yuqori ($\sim 2,9 \times 10^3$ siljish/ion), ZnO da o'rtacha ($\sim 3,1 \times 10^3$ siljish/ion) va ZnO:Mg da eng past ($\sim 2,6 \times 10^3$ siljish/ion) qiymatni ko'rsatdi. Bu esa Mg bilan legirlash nuqsonlar hosil bo'lish jarayonini samarali ravishda kamaytirishini anglatadi. Mg^{2+} ionlarining Zn^{2+} panjarasiga kiritilishi atom massasi va potensial maydonni o'zgartirib, panjaraning

barqarorligini oshirdi hamda energiya taqsimotini bir tekislashtirdi. Bu ZnO:Mg materialining radiatsiyaga chidamliligini sezilarli darajada yaxshiladi va nuqsonlarning chuqurlik bo'yicha tarqalishini kamaytirdi. To'xtatish qobiliyati ($-dE/dx$) yuqori energiyalarda asosan elektron to'xtatish jarayoni orqali boshqarildi, diapazon oxiri (EOR) mintaqasida esa yadroviya sochilish mexanizmlari ustunlik qildi. Shu sababli har bir material uchun energiya taqsimoti turidagi profilni hosil qildi. Shunday qilib, Mg bilan legirlangan ZnO yuqori energiyali zarrachalarga nisbatan eng barqaror va radiatsiyaga chidamli oksid yarimo'tkazgich sifatida samarali qo'llanishi mumkinligi aniqlandi. Olingan natijalar ion-nurlanish muhitida ishlovchi optoelektron va gaz sezgir qurilmalar barqarorligini oshirish uchun muhim ilmiy asos bo'lib xizmat qiladi.

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Tojiboev G'aybullo — Andijon davlat universiteti talabasi, Yarimo'tkazgichlar fizikasi laboratoriyasi muhandisi

Scopus Author ID: 57243482800, ORCID: 0009-0000-5022-8108.

Adixamov Nuriddin — Andijon davlat universiteti talabasi, Yarimo'tkazgichlar fizikasi laboratoriyasi muhandisi

Arabboyeva Mohira — Andijon davlat universiteti talabasi, Yarimo'tkazgichlar fizikasi laboratoriyasi muhandisi

Mualliflar haqida ma'lumot

Hotamjon Mansurov — fizika-matematika fanlari nomzodi; Andijon davlat universiteti Umumiy fizika kafedrasida dotsenti; ORCID: 0009-0006-4571-7795

Yunusaliyev Nuritdin — fizika-matematika fanlari bo'yicha falsafa doktori (PhD). Andijon davlat universiteti Umumiy fizika kafedrasida o'qituvchisi. Scopus Author ID: 59132890200, ORCID: 0000-0003-3766-5420;

DOPED p^0-n^0 HETEROSTRUCTURE SEMICONDUCTORS AND THE EFFECT OF AG DOPING ON THEM

A.Sultanov¹, X. Bozorov¹, F.Kasimov²,

¹ Namangan davlat texnika universiteti, O'zbekiston, Namangan sh,

² Z. M. Bobur nomidagi Andijon davlat universiteti, O'zbekiston, 170100, Andijan sh, Universitet ko'chasi 129 uy

2025-yil 2-sentyabrda berilgan; 2025-yil 29-oktyabrda chop etish uchun qabul qilingan

Annotatsiya. Galliy arsenid boshqa ko'plab A3B5 yarimo'tkazgichlar singari, o'ta tez elektron va optik qurilmalar uchun muhim materialdir, ammo ular deyarli har doim kristall panjarada nuqtaviy nuqsonlarni o'z ichiga oladi, ularning o'lchami panjara atomlarining o'lchamiga teng. Tajribalar o'tkazildi va Ag aralashmasining yuqori kuchlanishli p^0-n^0 geterostrukturalarga va yengil legerlangan galliy arsenid qatlamiga asoslangan qurilmalarga ta'siri o'rganildi. Ag aralashmalarining p^0-n^0 o'tish holatiga, ko'pchilik zaryad tashuvchilarning konsentratsiyasi va tarqalish tabiatiga, oz zaryad tashuvchilarning diffuziya uzunligiga, zaryad tashuvchilarning harakatchanligiga, yengil legerlangan galliy arsenid qatlamida chuqur sathlarning paydo bo'lishiga ta'siri o'rganildi.

Kalit so'zlar: Geterobirikmalar, yuqori kuchlanishli p^0-n^0 strukturalar, yengil legerlangan GaAs qatlamlari, aralashmalar, diffuziya uzunligi va zaryad tashuvchilarning harakatchanligi, chuqur sathlar.

Аннотая. Арсенид галлия, как и многие другие полупроводники A3B5, является важнейшим материалом для сверхбыстрых электронных и оптических приборов, но они почти всегда содержат точечные дефекты кристаллической решетки, размер которых равен размеру атомов решетки. Проведены эксперименты и изучено влияние Ag примеси на высоковольтных p^0-n^0 гетеропереходов и приборов на основе слаболегированного слоя арсенида-галлия. Исследовано влияние примеси Ag на место положения p^0-n^0 перехода, определена концентрация и характер распределения основных носителей заряда, диффузионная длина неосновных носителей заряда, подвижности носителей заряда, возникновения глубоких уровней в слаболегированного слоя арсенида-галлия.

Ключевые слова: Гетеропереходы, высоковольтные p^0-n^0 переходы, слаболегированные слои GaAs, примеси, диффузионная длина, и подвижности носителей заряда, глубокие уровни.

Abstract. Gallium arsenide, like many other A3B5 semiconductors, is an essential material for ultrafast electronic and optical devices, but they almost always contain point defects in the crystal lattice, the size of which is equal to the size of the lattice atoms. Experiments have been carried out and the influence of Ag impurity on high-voltage p^0-n^0 heterojunctions and devices based on a lightly doped gallium arsenide layer has been studied. A study of the influence of Ag impurities on the position of the p^0-n^0 transition, the concentration and nature of the distribution of the majority charge carriers, the diffusion length of minority charge carriers, the mobility of charge carriers, the occurrence of deep levels in a lightly doped gallium arsenide layer were determined.

Keywords: Heterojunctions, high-voltage p^0-n^0 junctions, lightly doped GaAs layers, impurities, diffusion length, and charge carrier mobility, deep levels.

I. INTRODUCTION.

Carrying out research [1,2] on the creation of powerful high-voltage switching device structures based on lightly doped gallium arsenide is associated with the need to search for alternative principles for switching electrical power in the subnano- and picosecond time ranges [3,4,5]. Currently, subnano- and picosecond pulse technology is developing extremely rapidly, which is associated with the needs of laser ranging and radio transmitting technology; thermonuclear energy, picosecond spectroscopy of liquids and solids, topography and a number of areas of conversion technology, as well as physical experiment

techniques require the creation of semiconductor switches this power range, which has the traditional advantages of semiconductor devices: long service life, reliability, high efficiency. and resistance to external influences and instant readiness for work [6]. An urgent task is to obtain epitaxial layers and devices based on A3B5 compounds (GaAs, GaP) with specified and new properties, in which the compensation of layers, the distribution of the concentration of background small impurities and defects with deep levels (DL) are determined by the technological conditions of melt crystallization. At the time of the work, there were practically

no experimental studies on the influence of the content of isovalent Ag impurity in the liquid phase during the growth of GaAs epitaxial layers on the concentration and type of intrinsic lattice defects formed in this case, and there was no sufficient understanding of the role and nature of acceptor defects with deep-dose, as well as donor type EL2 defects involved in compensation of such a structure.

An urgent problem in the physics of defects and semiconductors is the study of the metastability of defects and their properties, primarily the EL2 defect and the DX center in GaAs and AlGaAs, as well as other defects formed during epitaxial growth and during their radiation irradiation with protons and electrons. The question is what is an EL2 defect - an isolated antisite defect or its complex with intersite arsenic, whether it can be formed in GaAs grown from a solution-melt in Ga and under radiation irradiation or not, just like DX - the center is a defect with a negative correlation energy, and if so, then whether it can go into the metastable antibound state A_i of the substitution donor was open until recently. Very urgent problems are the identification of defects in complex layered structures and the study of physical characteristics in local areas of semiconductor materials and devices. Electron probe research methods meet these requirements. By the time work began on the study of materials and structures based on epitaxial undoped GaAs, there were not sufficient adequate electron probe methods for determining the parameters of defects and impurities in local areas of materials and devices, as well as the uniformity in their distribution. The need to develop such methods has become an urgent task. The purpose of this work is to grow, experimentally study and identify new properties and nature of lattice point defects with deep levels in epitaxial layers and structures based on A₃B₅ compounds (such as GaAs, $\text{Ga}_{1-x}\text{Al}_x\text{As}$ and $\text{Ga}_{1-x}\text{In}_x\text{As}$), formed by various methods of growth, annealing and radiation irradiation of layers. Very urgent problems are the identification of defects in complex layered structures and the study of physical characteristics in local areas of semiconductor materials and devices [7,9]. Electron probe research methods meet these requirements. By the time work began on the

study of materials and structures based on epitaxial undoped GaAs, there was not sufficient evidence of adequate electron probe methods for determining the parameters of defects and impurities in local areas of materials and devices, as well as the uniformity in their distribution. The need to develop such methods has become an urgent task.

II. METHOD.

It is of interest to study the influence of isovalent atoms on the process of formation of smooth p₀-n₀ transitions, since their use makes it possible to change the concentration level and spectrum of small impurities and deep-level traps due to interaction with background impurities in the melt and changes in intrinsic point defects in epitaxial layers. Epitaxial growth methods make it possible to combine in time the process of crystallization of a semiconductor material and the production of a device structure. Gas transport, molecular beam and liquid phase epitaxy (LPE) have become the most widespread.

Deal joining of crystal lattices into a semiconductor heterojunction is possible using LPE. This method makes it possible to produce not only two-layer but also multilayer heterostructures. Understanding the role of impurities and the resulting defects is of decisive importance for explaining a number of phenomena from diffusion to one-sided injection, or for constructing a theory of processes occurring in alloyed materials. To achieve these goals, the following set of tasks was solved [8,10,13]. Research and determination of the influence of Ag impurity on the main physical parameters of a lightly doped gallium arsenide layer, such as: location of high-voltage p₀-n₀ junctions; -diffusion lengths of minority charge carriers and their mobility in lightly doped gallium arsenide layers; -optimization of the technology for producing layers and structures with controlled defect content to create new, efficient micro- and optoelectronics devices;

-Development of electron probe methods for studying the process of recombination and spectroscopy of centers with deep levels in local areas of epitaxial layers, based on recording the current induced by an electron probe.

- Determination of the energy spectrum of deep-level lattice defects and quantum states of dots, as well as analysis of the influence on this spectrum of both external and built-in electric fields using ELTB spectroscopy under various conditions of preliminary reversible isochronous, isothermal and optical annealing.

-Application of capacitance-voltage measurements and transient spectroscopy of deep-level spectroscopy to determine the degradation mechanism of quantum well lasers. It has been shown that both an increase in the crystallization onset temperature during the growth of GaAs from a solution-melt in Ga and a change in the Ag content in the liquid phase make it possible to control the concentration and range of intrinsic acceptor (HL2, HL5 and HF1) and donor (EL2) point defects, compensation of layers and structures [10,13]. Shown it has been shown that in situ annealing of AlGaAs layers promotes the formation of dislocation-free coherent QDs, the annihilation of point lattice defects associated with the formation of dislocations in the GaAs matrix layer (EL2), localized at the AlGaAs/GaAs heterointerface, and a decrease in the concentration of other defects by an order of magnitude. A new scientific direction of research that was formed in the process of carrying out this work is the study of new properties and the nature of point lattice defects with deep levels in epitaxial layers and structures based on A3B5 compounds (such as GaAs and GaAlAs), the effects of their interaction with self-organizing Ga/GaAs quantum dots using new methods for studying centers with deep levels in local regions of layered structures.

II. EXPERIMENTAL.

To solve the problems, the following methods were used: After

growing the epitaxial structures, the thickness of the layers, their planarity and the location of the p⁰-n⁰ transition were first determined [1]. To determine the location of the p⁰-n⁰ junction or space charge region (SCR), the electro-optical effect method was used [12]. The electro-optical effect manifests itself in the occurrence of double beam penetration in an electric field. In the case of an epitaxial structure with a p⁰-n⁰ junction, double beam interference should occur in the SCR due to the indication of optical anisotropy at the p⁰-n⁰ junction. During the studies, a MIC-5 microscope was used.

To determine the concentration and nature of the distribution of the main charge carriers, the capacitance-voltage method was used using a mercury probe during layer-by-layer etching of gallium arsenide layers. A typical distribution of charge carrier concentration in one of the resulting p⁰-n⁰ structures is shown in Fig.1.

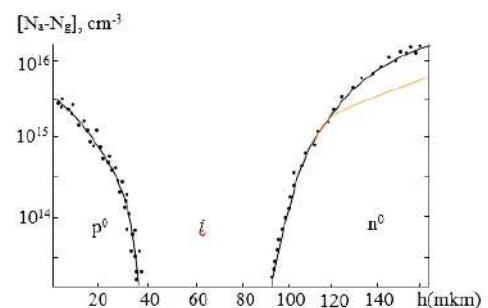


Fig 1. Concentration distributions charge carriers in one p⁰-n⁰ structures.

The diffusion length of the NCC is the most important characteristic of any semiconductor material [6,11,14]. To determine the diffusion length of the NCC in lightly doped GaAs layers introduced with Ag impurities, a method was used to study the current distribution induced by a small-diameter electron probe and is determined by the expression

$$I = I_0 \exp \left(- \frac{r_0 - x}{L_d} \right)$$

where x - is the distance from the excitation area to the p⁰-n⁰ transition, r_0 is the radius of the excitation area.

The diffusion lengths L_d in n⁰ - GaAs, determined by this method from the slope of the

dependence $L_{n1} = f(x)$, reached $6 \div 15 \mu\text{m}$ at zero bias and increased to $\geq 80 \mu\text{m}$ at $U_{\text{arr}} = 100 \text{ V}$.

The measurements were carried out using a jXA-5A scanning electron microscope microanalyzer. Figure 2 shows the dependence of the diffusion length of electrons on the current in a lightly doped p^0 – GaAs layer with a thickness of 35 microns. It can be seen that starting with current densities $j > 50 \text{ A/cm}^2$, the diffusion length L_n practically does not change and reaches values of $75 \div 85 \mu\text{m}$. The increase in L_n in the region of lower currents is apparently associated with the saturation of electron capture centers.

The charge carrier mobilities were determined by measuring the Hall effect in epitaxial layers grown on semi-insulating substrates.

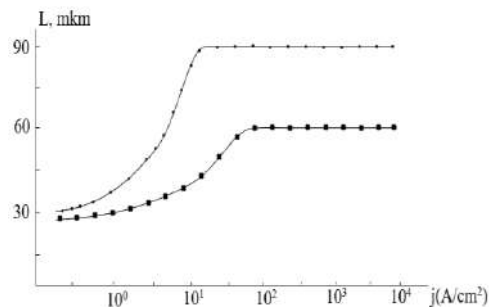


Fig 2. Typical dependences $L = f(j)$ in p^0 = GaAs layers. 1 – $p^0 = 5 \cdot 10^{15} \text{ cm}^{-3}$; 2 – $p^0 = 2 \cdot 10^{15} \text{ cm}^{-3}$.

The measurements were carried out by the van der Pauw method on samples with six ohmic contacts obtained by fusing Indian drops into the epitaxial layer. The values of electron mobility in n^0 -GaAs layers are in the range $\approx 5000 \div 6000 \text{ cm}^2/\text{V} \cdot \text{sec}$, and the hole mobility in p^0 - n^0 -GaAs layers is $\approx 3000 \div 5000 \text{ cm}^2/\text{V} \cdot \text{sec}$.

The results of studies of the mobility of the main charge carriers in the n^0 -regions showed that there is a high degree of compensation in epitaxial layers Fig. - 3. As follows from the figure, the degree of compensation in the n^0 layer increases as it approaches the p^0 - n^0 transition. This nature of the mobility dependence may indicate that the total concentration of charged centers in the layer decreases with increasing thickness of the n^0 -

layer and with decreasing crystallization temperature of the layer.

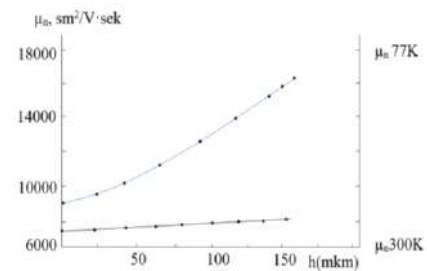


Fig 3. Electron mobility in the layer - lightly doped GaAs.

To determine the energy parameters of deep levels in lightly doped layers introduced with Ag impurities, the method of transient capacitive deep level spectroscopy (DLTS) and photocapacitance (PC) were used. In all layers obtained by the LPE method, in addition to shallow acceptor and donor levels, deep levels $A(E_c + 0.32 \text{ eV})$, $B(E_c + 0.54 \text{ eV})$, $C(E_c + 68 \text{ eV})$ were detected Fig. - 4. The crosssection for capturing deep levels corresponds to the value $\sigma = 8 \cdot 10^{-17} \text{ cm}^2$, $6 \cdot 10^{-13} \text{ cm}^2$, $4 \cdot 10^{-17} \text{ cm}^2$.

Unit rat.

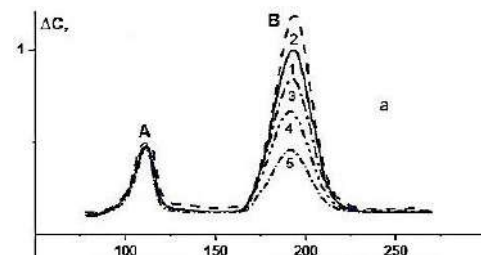


Fig 4. DLTS spectrum one of the samples doped with Ag impurity.

In the literature, these levels are designated HL5, HL3, and HL2 [3-4]. The recombination properties of the p^0 region are largely determined by the deep acceptor A and B centers. In this case, these centers play the main role in the p – part of the p^0 region.

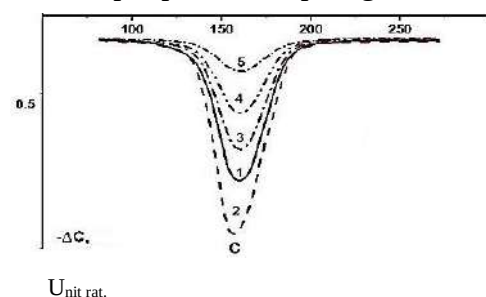


Fig 5. IR absorption spectrum doped with Ag at various operating temperatures.

p^0 -GaAs, they, detected by capacitance spectroscopy (DLTS) and attributed to a growth defect, are traps for holes, and their concentration decreases exponentially with decreasing growth temperature. The IR absorption spectrum of these samples doped with Ag impurities at temperatures from 20 °C to 200 °C are presented in the Fig. - 5, and the kinetics of the influence of temperature in the range from 20 °C to 200 °C at deep levels is shown in Fig. - 6.

Along with shallow acceptors and shallow donors, deep acceptors determine the position of the p^0 - n^0 junction and the size of the SCR. The part of the p^0 layer adjacent to the substrate has a relatively high concentration of small acceptors, greater than the concentration of small donors and sponge acceptors. As mixing progresses from the substrate deeper into the layer, the difference $N_{d.m.} - N_{m.a.}$ decreases and then becomes equal to zero and changes sign, however, due to the presence of deep acceptors (Fig. 6), the type of conductivity of the layer does not change.

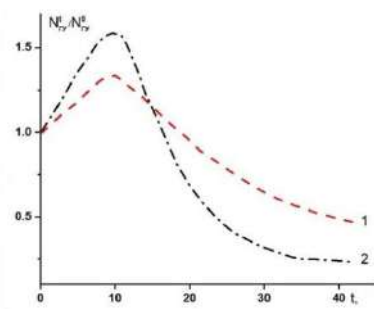


Fig 6. Kinetics of workers' influence temperatures on GI samples doped with Ag.

If $0 < N_{d.m.} - N_{m.a.} < N_{a.g.}$, then the Fermi level will be close to the position of the first deep acceptor level - A. With further displacement deeper into the layer, when $N_{a.g.} < N_{a.m.} - N_{a.m.} < N_{a.g.}$, the Fermi level will approach the position of the second level - B and further to the third level - C.

Estimates show that the resistivity of the region of non-field ionization of centers with levels A, B and C at 300 K is 10^6 Ohm·cm or more. Layer coordinate, where $N_{d.m.} = N_{a.m.} + 2N_{a.g.}$ is the location of the p^0 - n^0 transition. Further, in the n^0 -type layer, the electron concentration will be determined by a decrease in $N_{d.m.} - N_{a.g.}$. Therefore, the p^0 region consists of two parts: a low-resistance p^0 and a high-resistance i part, and the p^0 - n^0 junction is located on the border of the i and n^0 regions.

Scientific novelty of the work. All main scientific results were obtained for the first time.

- Methods have been developed for determining the optical parameters of centers with deep levels, recombination processes in local regions of p^0 - n^0 structures using electron probe and separation in the spectra of BLTB signals associated with quantum states of dots from defects with deep levels.

The properties, conditions for the formation and annealing of the EL2 defect and the NB1 acceptor defect with a level of $(E_y + + 0.47)$ eV in GaAs epitaxial layers grown from a melt solution in Ga at crystallization onset temperatures not exceeding 900°C were discovered and studied.

The properties and conditions for the formation of a bistable defect with the kinetics of first-order configurational transformation and parameters in a stable state coinciding with the radiation E1 defect, the transformation kinetics of which is determined by a single jump of interstitial arsenic, were discovered and studied; metastable level of the antibound localized A1 state of the OX center in doped AlGaAs layers.

In GaAs/AlGaAs heterostructures, the influence of defects with deep levels on the population of states of quantum dots has been studied; Coulomb interaction between ionized lattice defects and carriers localized in quantum dots, with the formation of an electrostatic dipole, the built-in field of which changes the height of the

potential barrier for the emission and capture of carriers by the quantum dot.

It has been shown that both an increase in the temperature of the onset of crystallization during the growth of GaAs from a solution-melt in Ga and a change in the Ag content in the liquid phase make it possible to control the concentration and nomenclature of intrinsic acceptor (HL2, HL5 and HF1) and donor (EL2) point defects, layer compensation and structures.

It has been shown that annealing of AlGaAs layers promotes the formation of dislocation-free coherent QDs, annihilation of lattice point defects associated with the formation of dislocations in the GaAs matrix layer (EL2), localized at the GaAs/AlGaAs/ heterointerface, and a decrease in the concentration of other defects by an order of magnitude.

IV. Conclusions.

In conclusion, we note that

- The positions of the p^0 - n^0 transition or the space charge region of the lightly doped GaAs layer doped with Ag impurities are determined. The thickness of the p^0 region is determined by the values of the transmission coefficient and the recombination properties of the resulting p^0 - n^0 transitions [11,12,14].

- The nature and profile of the concentration distribution of the main charge carriers have been studied. The carrier concentration in the p^0 - and n^0 -regions ranged from $1 \cdot 10^{15}$ to $0.1 \cdot 10^{15} \text{ cm}^{-3}$.

The mobility of charge carriers is determined. The mobility of charge carriers was, in the n^0 - layer $\mu_n \approx (5 \div 6) \cdot 10^3 \text{ cm}^2/\text{V} \cdot \text{sec}$, and in the p^0 - layer $\mu_p \approx (400 \div 450) \text{ cm}^2/\text{V} \cdot \text{sec}$.

- The diffusion lengths of the NCC in lightly doped GaAs layers doped with Ag impurities were determined;

A comprehensive study of the properties of a lightly doped GaAs layer doped with Ag impurities was carried out and it was found that these impurities form deep levels

with ionization energies $E_c - 0.32 \text{ eV}$, $E_c - 0.82 \text{ eV}$, $E_v - 0.55 \text{ eV}$ in GaAs <Ag>.

- The influence of deep centers created by Ag impurities has been studied and it has been established that impurities are unstable in the gallium arsenide lattice when operating temperatures change and the kinetics of the influence of temperature on deep levels is nonlinear.

Levels that are traps for electrons ($E_c - 0.53 \text{ eV}$), ($E_c - 0.82 \text{ eV}$) and ($E_v - 0.55 \text{ eV}$), designated as EL2, EL3, EL4, were also discovered in n^0 -GaAs layers. The capture cross section is $6 \cdot 10^{-14} \text{ cm}^2$, $5 \cdot 10^{-17} \text{ cm}^2$ and $7 \cdot 10^{-14} \text{ cm}^2$, respectively. The concentration of the EL3 trap was about 10^{17} cm^{-3} [10].

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Mualliflar haqida ma'lumot

1. Sultanov Ahmadjon Majidovich – f.-m.f.n., dotsent, Namangan davlat texnika universiteti, dotsenti.

E-mail: sulix80@mail.ru

2. Bozorov Xurshid Narzullayevich – f.-m.b.PhD., dotsent, Namangan davlat texnika

universiteti, dotsenti. E-mail: xurshid_bozorov83@mail.ru ;

3. Kasimov Farrux Unvanovich – f.-m.b.PhD., Andijon davlat universiteti, Kondensirlangan muhitlar fizikasi kafedrası, katta o'qituvchi. E-mail: foruq7777@gmail.com.

GaAs_{1-δ}Bi_δ kvant o'rali (ZnSe)_{1-x-y}(Ge₂)_x qattiq qorishmlarning tuzilmaviy xossalariga gamma nurlanishlar ta'siri

S.Z.Zaynabidinov¹ A.Y.Boboyev¹, Sh.X. Yo'ldiyev², I.M.Soliyev²

¹Z.M.Bobur nomidagi Andijon davlat universiteti, 1700100, Andijon, O'zbekiston

²Andijon davlat pedagogika instituti, Andijon, O'zbekiston

E-mail: igboljonsoliyev45@gmail.com

2025-yil 5-avgustda berilgan; 2025-yil 29-oktyabrda chop etish uchun qabul qilingan

Annotatsiya: Ushbu tadqiqotda suyuq fazali epitaksiyalanishning majburiy sovitish tizimida (100) yo'nalishli n-GaAs tagliklarda o'stirilgan (ZnSe)_{1-x-y}(Ge₂)_x(GaAs_{1-δ}Bi_δ)_y qattiq qorishmalarining rentgen-tuzilmaviy xususiyatlariga 10⁵–10⁶ rad dozali gamma nurlanishlarining ta'siri eksperimental jihatdan o'rganildi. Rentgen mikroanaliz natijalari epitaksial qatlam bo'ylab ZnSe, Ge₂ va GaAs_{1-δ}Bi_δ komponentlarining notekis taqsimlanishini, shuningdek, kristallanish jarayonida Ge va GaAsBi atomlarining yuqori to'yinganlik sohalarida to'planishini ko'rsatdi. Rentgen difraksiya tahlillari (100) yo'nalishga ega sfalerit tuzilishli monokristall hosil bo'lganini, gamma nurlanish dozasi ortishi bilan kristall panjarada mikrokuchlanishlar, subkristallit o'lchamlarining o'zgarishi va asosiy difraksiya chiziqlarining siljishini aniqladi. Tadqiqot natijasida 21–27 nm o'lchamli Ge nanokristallari va GaAsBi asosidagi kvant o'ralarining shakllanishi, ularning 10⁵ gamma nurlanish ta'sirida nisbatan barqarorlashuvi oshib mukammallik darajasi ortgani hamda 10⁶ rad dozali gamma nurlanish ta'sirida nuqsonlarni ortishi natijasida mukammallik darajasi kamaygani aniqlangan. Olingan natijalar epitaksial ko'p qatlamli yarimo'tkazgich tuzilmalarining optik va elektr xossalarini boshqarishda muhim ahamiyatga ega.

Kalit so'zlar: Epitaksiya, qattiq qorishma, (ZnSe)_{1-x-y}(Ge₂)_x(GaAs_{1-δ}Bi_δ)_y, rentgen difraksiyasi, kristall panjara, gamma nurlanishi, nanokristallitlar, kvant o'ralar, sfalerit tuzilma, subkristallit o'lchami.

Аннотация: В данной работе экспериментально исследовано влияние гамма-излучения с дозами 10⁵–10⁶ рад на рентгеноструктурные свойства твердых растворов (ZnSe)_{1-x-y}(Ge₂)_x(GaAs_{1-δ}Bi_δ)_y, выращенных методом жидкофазной эпитаксии с принудительным охлаждением на подложках n-GaAs ориентации (100). Результаты рентгеновского микроанализа показали неравномерное распределение компонентов ZnSe, Ge₂ и GaAs_{1-δ}Bi_δ по толщине эпитаксиального слоя, а также локализацию атомов Ge и GaAsBi в высоконасыщенных областях в процессе кристаллизации. Анализ рентгеновской дифракции выявил формирование монокристалла со sfalerитной структурой и ориентацией (100), а также установил, что с увеличением дозы гамма-излучения возникают микронарушения кристаллической решётки, изменяются размеры субкristаллитов и наблюдаются смещения основных дифракционных линий. В ходе исследования выявлено образование Ge нанокристаллов размером 21–27 нм и квантовых ям на основе GaAsBi, а также установлено, что под воздействием дозы 10⁵ рад их устойчивость и степень совершенства увеличиваются, тогда как при дозе 10⁶ рад рост концентрации дефектов приводит к снижению совершенства структуры. Полученные результаты имеют важное значение для управления оптическими и электрическими свойствами многослойных эпитаксиальных полупроводниковых гетероструктур.

Ключевые слова: Эпитаксия, твердый раствор, (ZnSe)_{1-x-y}(Ge₂)_x(GaAs_{1-δ}Bi_δ)_y, рентгеновская дифракция, кристаллическая решётка, гамма-излучение, нанокристаллиты, квантовые ямы, sfalerитная структура, размер субкristаллитов.

Abstract: This study experimentally investigates the effect of gamma irradiation with doses of 10⁵–10⁶ rad on the X-ray structural properties of (ZnSe)_{1-x-y}(Ge₂)_x(GaAs_{1-δ}Bi_δ)_y solid solutions grown by liquid-phase epitaxy with forced cooling on (100)-oriented n-GaAs substrates. X-ray microanalysis revealed a nonuniform distribution of ZnSe, Ge₂, and GaAs_{1-δ}Bi_δ components throughout the epitaxial layer, as well as the accumulation of Ge and GaAsBi atoms in highly saturated regions during crystallization. X-ray diffraction analysis confirmed the formation of a (100)-oriented sphalerite monocrystal and showed that increasing gamma irradiation dose leads to lattice microstrains, variations in subcrystallite sizes, and shifts in the main diffraction peaks. The study also identified the formation of 21–27 nm Ge nanocrystals and GaAsBi-based quantum wells, establishing that exposure to 10⁵ rad gamma irradiation enhances their structural stability and perfection, whereas a dose of 10⁶ rad increases defect concentration, thereby reducing structural perfection. These results are significant for controlling the optical and electrical properties of multilayer epitaxial semiconductor heterostructures.

Keywords: Epitaxy, solid solution, $(\text{ZnSe})_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$, X-ray diffraction, crystal lattice, gamma irradiation, nanocrystallites, quantum wells, sphalerite structure, subcrystallite size.

KIRISH.

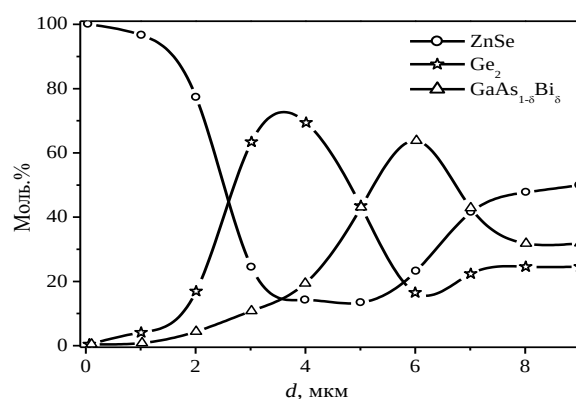
Ma'lumki, yarimo'tkazgichli nano - va mikroelektron tizimlarining funksional imkoniyatlarini kengaytirish kengaytirish bilan bog'liq bir qator ilmiy-texnik muammolarni hal qilish bugungi kunning dolzarb muammolaridan biridi. Hozirgi vaqtda ushbu muammolarga bag'ishlangan yetarli miqdordagi nashrlar mavjudligiga qaramay, Si, Ge, GaAs va boshqa yarimo'tkazgichli monokristallarning energetik soha kengligida turli xil energetik sathlarini xosil qiladigan kirishma atomlari bilan legirlash, shuningdek o'stirish paytida kiritilgan nuqsonlar va gamma nurlari yordamida qisman faza almashtirish, ham yarimo'tkazgichli materiallarda, ham yarimo'tkazgichli ko'p qatlamli tuzilmalarda bunday nuqsonlarning xatti-harakati va tok o'tishiga ta'sirining o'ziga xos xususiyatlari bo'yicha muammolar yetarlicha. Shu bilan birga, epitaksial yupqa plyonkalarining qotishma va mukammal tuzilmalarini yaratish uchun maqbul texnologik sharoitlarni aniqlash, shuningdek, ularning xususiyatlarini maqsadli boshqarishga imkon beradigan zarur komponentlar va kirishma atomlarini tanlash hamda ularga turli tashqi ta'sirlarni tadqiq qilish katta ilmiy va amaliy qiziqish uyg'otadi [1]. Shu bilan birga molekulyar kirishmalarni o'rganish natijalari, qattiq qorishmadagi nanokristallitlarning harakati, tok o'tish jarayonlarining asosiy material tarkibiga bog'liqligi, ayniqsa yarimo'tkazgichli materiallarda shakllangan nanokristallitlarga tashqi ta'sirlar haqida ma'lumotlar hali ham yetarli emas.

Shu munosabat bilan tadqiqot ishining ushbu bo'limida suyuq fazali epitaksiyalanishning majburiy sovutish tizimda vismut hosil qiluvchi aralashma eritmadan (100) kristallografik tartibli $n\text{-GaAs}$ tagliklariga o'stirilgan $\text{GaAs}_{1-\delta}\text{Bi}_\delta$ kvant o'rali $(\text{ZnSe})_{1-x}(\text{Ge}_2)_x$ qattiq qorishmlarini rentgenotuzilmaviy xususiyatlariga gamma nurlanishlar ta'sirini o'rganish bo'yicha eksperimental natijalar keltirilgan.

TADQIQOT METODI.

"Jeol" JSM 5910 LV rentgen mikroanalizatori yordamida o'stirilgan $(\text{ZnSe})_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$

epitaksial qatlamlarning kimyoviy tarkibi o'rganildi. Rentgen mikroanalizatorining tahlil qilish natijalariga ko'ra, $(\text{ZnSe})_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$ epitaksial qatlamning qalinligi bo'yicha ZnSe Ge_2 ba $\text{GaAs}_{1-\delta}\text{Bi}_\delta$ birikma molekularining taqsimlanish profili aniqlangan (1-rasm). Rasmdan $(\text{ZnSe})_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$ epitaksial qatlamning qalinligi ortishi Ge_2 va $\text{GaAs}_{1-\delta}\text{Bi}_\delta$ molyar miqdorlari dastlab asta-sekin o'sib boradi va mos ravishda eng yuqori $x=0,725$ ba $y=0,638$ qiymatlariga erishganini ko'rish mumkin. Bu esa o'z navbatida kristallanish sohasida aralashma eritma Ge_2 va $\text{GaAs}_{1-\delta}\text{Bi}_\delta$ larga yuqori to'yinganligidan dalolat beradi. Shuningdek, Ge_2 ba $\text{GaAs}_{1-\delta}\text{Bi}_\delta$ molyar miqdorlari asta sekinlik bilan kamayib, epitaksial qatlam sirtga yaqin sohalarda $x=0,23$ ba $y=0,3$ qiymatlariga erishadi. Epitaksial qatlamning o'sishi chegeralangan hajmdagi aralashma eritmada amalga oshirilganligi uchun suyuq vismutdagi ZnSe eruvchanligidan $\text{GaAs}_{1-\delta}\text{Bi}_\delta$ eruvchanligi 3 marta, Ge eruvchanligi esa 2 marta kamligi tufayli Ge va $\text{GaAs}_{1-\delta}\text{Bi}_\delta$ molyar miqdorlarining o'sish yo'nalishi bo'yicha asta-sekin kamayishiga olib keladi [2]. Plyonkaning sirtining 1 mkm chuqurligidan boshlab Ge va $\text{GaAs}_{1-\delta}\text{Bi}_\delta$ ning molyar miqdori mos ravishda 23% va 30% dan oshmaydi. Shunday qilib, o'stirilgan epitaksial Plyonkalar o'zida tashkillovchilarining molyar miqdori ($0 \leq x \leq 0.725$ ba $0 \leq y \leq 0.638$) asta-sekin o'zgarib boradigan, o'rin almashinuvchi $(\text{ZnSe})_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$ qattiq qorishmlarini namoyon qilishi aniqlandi. Taglik va Plyonkaning sirtiy sohalari o'rtasida tor energetik sohali Ge va $\text{GaAs}_{1-\delta}\text{Bi}_\delta$, bilan boyitilgan qatlam shakllanishi aniqlandi.



1- rasm. $(\text{ZnSe})_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$ epitaksial qatlamning qalinligi bo'yicha ZnSe Ge_2 va $\text{GaAs}_{1-\delta}\text{Bi}_\delta$ birikma molekularining taqsimlanish profili $\text{GaAs}_{1-\delta}\text{Bi}_\delta$ kvant o'rali $(\text{ZnSe})_{1-x}(\text{Ge}_2)_x$ qattiq qorishmlarini rentgenotuzilmaviy tadqiqotlari

Empyrean Malvern turidagi uchinchi avlod rentgen difraktometri yordamida o'rganilgan. Rentgen difraksiyasini o'lchash Bregg–Brentano nurlari geometriyasida $2\theta\text{B} = \text{ot } 15^\circ \text{ do } 100^\circ$ diapazonda amalga oshirildi va skanerlash tezligida doimiy ravishda 1 gradus/min ni tashkil etdi. Eng yuqori kuztilgan tuzilmaviy cho'qqini aniqlash uchun OriginPro2019 dasturi ishlatilgan

TADQIQOT NATIJALARI.

2-rasmda $(\text{ZnSe})_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$ epitaksial qatlamining rentgengrammasi keltirilgan (qora rangli egri chiziq) va unda (100) kristallografik oriyentatsiyaga xos bo'lgan $\{H00\}$ (bu yerda $N=1, 2, \dots$) turidagi tanlanma tuzilmaviy akslanishlar seriyasi kuzatilgan. Asosiy tuzilma chizig'ining α_1 va α_2 nurlanish tashkillovchilariga sezilarli ajralishi, lekin ular intensivliklarning $(I(\alpha_1) = 2 I(\alpha_2))$ nisbatda bo'lishi plyonkanning sirtiy sohalaridagi kristall panjarada kichik miqdorda elastik mikrokuchlanishlar borligini ko'rsatadi. Panjara buzilishining bunday joylari foydalanilayotgan taglik kristall панжара доимийсининг ошишига olib keladi, u Nelson–Reyli tomonidan ekstrapolyatsiya funksiyasidan foydalangan holda [3]

$$f = (1/2) \cdot [(\cos^2\theta/\theta + (\cos^2\theta/\sin\theta))], \quad (1)$$

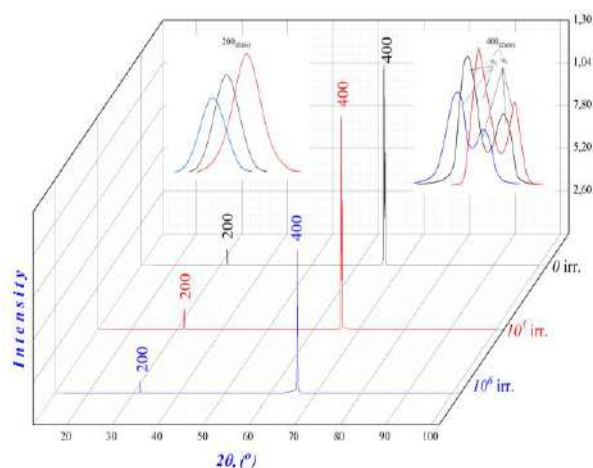
asosiy tuzilmaviy akslanishning (400) eksperimental qiymatlari asosida aniqlanganda $a_{\text{K.K.}} = 0,5664 \text{ nm}$ ni tashkil etdi, va bu qiymat $a_{\text{GaAs}} = 0.5646 \text{ nm}$ jadvaldagi qiymatida ortiq va $a_{\text{ZnSe}} = 0.5661 \text{ nm}$ ga yaqin [109], (bu yerda θ – sochilish burchagi ($2\theta\text{B}$ difraksiya burchagining yarmi). Bu esa yuqoridagi keltirilgan tarkibiy natijalarni yana bir bor tasdiqlaydi, ya'ni shakllangan epitaksial plyonkalarining qariyb yarmini ZnSe molekulari tashkil qiladi. Quyidagi ifoda yordamida (400) tuzilmaviy akslanishlarning yarimkengligidan plyonkani tashkillovchi bloklar (subkristalitlar) ning o'lchami aniqlandi [4]:

$$D = K\lambda/(\beta\cos\theta), \quad (2)$$

Bu yerda D – kristallit o'lchami, λ – rentgen nurining to'lqin uzunligi, bizning hol uchun 0.154 nm ga teng, θ – sochilish burchagi ($2\theta\text{B}$ difraksiya burchak yarmi), β – rentgenogrammadagi chiziqlarning fizik kengligi (akslanishning yarim kengligi), $K \approx 0,94$ koeffitsiyent [5].

$$\beta = \frac{1}{2}(B - b + \sqrt{B(B - b)}) \quad (3)$$

Bu yerda B – akslanishning ko'rinma kengligi, b – akslanishning ko'rinma geometrik kengligi. Yuqoridagi formuladan foydalangan holda D qiymatlarini hisoblash shuni ko'rsatdiki, qattiq qorishma subkristalitlarining o'lchami 40 nm ni tashkil etdi. Shunday qilib, asosiy (400) tuzilmaviy akslanishning yarimkengligi ($\text{FWHM} = 4.5 \cdot 10^{-3} \text{ рад}$) va yuqori intensivligi ($1,2 \cdot 10^5 \text{ imp(сек}^{-1})$), shuningdek, rentgengramma noelastik fon sathining monoton xarakterga egaligi o'stirilgan plyonkalar kristall panjarasining mukammallik darajasi yetarlicha yuqori ekanligidan dalolat beradi.



2-rasm. Nurlantirilmagan va nurlantirilgan $(\text{ZnSe})_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$ epitaksial qatlamining rentgengrammalari

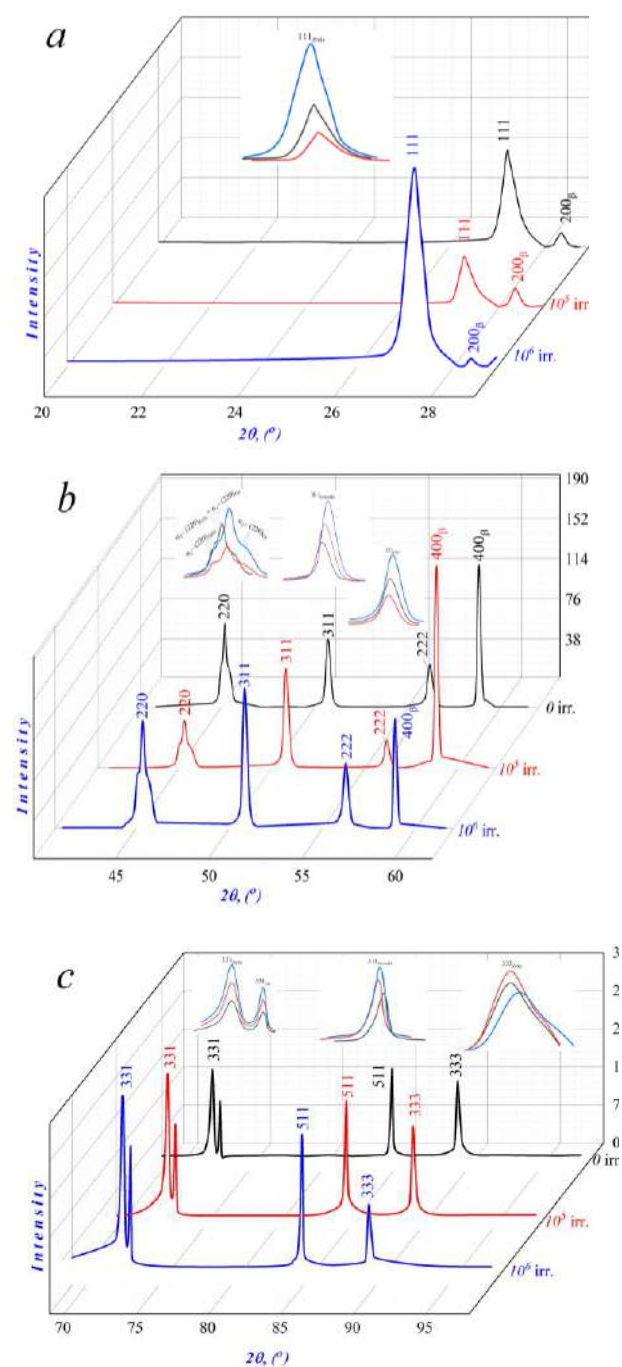
Shuningdek, 10^5 radli gamma nurlari bilan nurlantirilgan $(\text{ZnSe})_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$ epitaksial qatlamining rentgengrammasi ham keltirilgan (2-rasm, qizil rangli egri chiziq). Unda ham tanlanma, ya'ni yuqori intensivlikdagi tuzilmaviy akslanishlar nisbatan yuqori burchaklar tomon siljigan hamda intensivligi ortgan holda kuzatilgan. Kuzatilayotgan o'zgarishlar $(\text{ZnSe})_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)$ qattiq qorishma kristall panjarasining qiymati o'zgarayotganidan ($a_{\text{K.K.}}$

= 0,566 nm) va nisbatan kristall panjara barqorlashayotganidan dalolat beradi. Bu esa o'z navbatida nurlanish ta'sirida subkristallitlarni bo'lini chegarlarida kristall panjaraga elastik kuchlanish berayotgan kirishma atomlar qo'zg'alib turli yuqori potentsialli nuqsonga moyil sohalarda to'lanayotganini tasdiqlaydi. Shuningdek, bunday kirishma atomlarning kristall panjaralarda bo'linish chegaralari tomon qo'zg'alishlari epitaksial plyonkalarni tashkillovsi subkristallitlarning o'lchamlarini ortishiga ((2) ifoda yordamida aniqlangan 43.2 nm o'lcham) olib keladi []. Shunday qilib, $(\text{ZnSe})_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$ epitaksial qatlami 10^5 radli gamma nurlari bilan nurlantirilganda plyonkaning kristall panjarasida barqororlashuv yuzaga kelib, uning mkammallik darajasi ortdi [6].

10^6 radli gamma nurlari bilan nurlantirilgan $(\text{ZnSe})_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$ epitaksial qatlamining rentgenogrammasi ham keltirilgan (2-rasm, ko'k rangli egri chiziq). Unda kuzatilgan tanlanma tuzilmaviy akslanishlar quyi burchaklar tomon siljigan hamda ularning intensivliklari kamaygan. Mazkur holat uchun (400) tuzilmaviy akslanishning eksperimental qiymatlari yordamida 10^6 radli gamma nurlari bilan nurlantirilgan qattiq qorishmalarning kristall panjarasining qiymati $a_{q.q.} = 0,5667$ nm ekanligi aniqlandi. Bu esa o'z navbatida yuqori energiya nurlanishlar ta'sirida plyonkaning subkristallitlarning bo'lini chegarlarida hosil bo'lgan kristall panjarada kirishma atomlarning miqdori ortayotganidan dalolat beradi. Shuningdek, asosiy akslanishlar intensivligini kamayishi esa kristall panjaraning mukammallik darajasi tushib ketayotganligi haqida guvohlik beradi. Bundan tashqari (2) ifoda yordamida epitaksial plyonkalarni tashkillovsi subkristallitlarning o'lchamlarining qiymati 37.9 nm ekanligi aniqlandi. o'lcham) olib keladi [7]. Shunday qilib, 10^6 radli gamma nurlar $(\text{ZnSe})_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$ epitaksial qatlamda nuqsonlarni otishi natijasida subkristallitlarning o'lchamlari kamayishiga, shuningdek, kristall panjaraning mkammallik darajasi kamayishiga olib kelishi aniqlandi.

O'stirilgan epitaksial plyonkalar rengenogrammalarining $2\theta = 27,2^\circ$, $2\theta = 56,1^\circ$ va $2\theta = 89,6^\circ$ burchakli sochilishlarida mos

ravishda $d/n = 0,3271$ nmda $(111)_{\text{ZnSe}}$, $d/n = 0,1636$ nmda $(222)_{\text{ZnSe}}$ va $d/n = 0,2831$ nmda $(333)_{\text{ZnSe}}$ kristallografik oriyentatsiyali kuchsiz tuzilmaviy akslanishlar kuzatilgan (3-a, b va c rasmlarga qarang). ZnSe asosidagi sfalerit tuzilmalari uchun (100) kristallografik yo'nalishga ega monokristallarning yo'q bo'lib ketish qonunlariga muvofiq (111), (222) va (333) turdagi tuzilmaviy chiziqlar kuzatilishi tasdiqlanadi [8] va bu plyonkaning kristall panjarasida mikro buzilishlar paydo bo'lishini ko'rsatadi.



3-rasm. Nurlantirilmagan va nurlantirilgan $(\text{ZnSe})_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$ epitaksial qatlamining kichik (a), o'rta (b) va yuqori (c)

burchakli sochilishlardagi rentgenogramma shakllari.

Shuning mazkur akslanishlarning intensivliklar nurlanish dozasi ortganida o'zgargan. Bu esa o'z navbatida Plyonkaning kristall panjarasida nurlanishlar ta'sirida krishma atomlarining joylashuvi o'zgarishi natijasida turli elastik energiyalarni to'planib mikrokuchlanishlar hosil bo'layotganidan dalolat beradi. Kristall panjaraning bunday mikrobuzilishlarining mavjudligi plyonkada o'z-o'zidan shakllangan nanoobektlarning o'lchamlari o'zgarishiga sabab bo'lishi mumkin. Plyonka rentgenogrammasining $2\theta = 45.3^\circ$ burchakli sochilishlarida $d/n = 0.2001$ nmli tekisliklarda (220) hamda $2\theta = 71.4^\circ$ burchakli sochilishlarida $d/n = 0.1302$ nmli tekisliklarda (331) tartiblanishli tuzilmaviy akslanishlar ham kuzatilgan va gamma nurlanishlar ta'sirida ularning intensivliklari qisman o'zgargan. Bu esa o'z navbatida ZnSe molekulalaridan tashkil topgan subkristallitlarning bo'linish chegaralarida ortiqcha germaniy atmolari to'planib o'z-o'zidan $21 \div 27$ nm o'lchamdagi nanoobyektlarning shakllantirishidan dalolat beradi. Mazkur nanoobyektlarning panjara parametri (1) formulasi yordamida eksperimental ravishda aniqlandi va u $a_{Ge} = 0.5659$ nm bo'lib, bu qiymat ham uning jadval qiymatiga $a_{Ge} = 0.5657$ nm yaqin. $(ZnSe)_{1-x}-(Ge_2)_x(GaAs_{1-\delta}Bi_\delta)_y$ epitaksial qatlamlar 10^5 va 10^6 radli gamma nurlar bilan nurlantirilgan esa germaniy nanoobyektlarini o'lchamida va kristall panjara qiymatlari deyarli o'zgarish bo'lmagan. Shunday qilib, juftlashgan Ge atomlari plyonkaning matritsa panjarasining nuqsonga moyil sohalarida ZnSe molekulalarini qisman o'rin almashinadi va qolgan atomlar plyonka subkristallarining bo'linish chegaralarida $21 \div 27$ nm o'lchamdagi germaniy nanokristallarini shakllantirishda ishtirok etadi[9]. Rentgenogrammaning $2\theta = 50.9^\circ$ va $2\theta = 84.8^\circ$ burchakli sochilishlarida yana ikkita $d/n = 0.1791$ nm da (311) va $d/n = 0.1143$ nm da (511) tartiblarida kuchsiz tuzilmaviy akslanishlar kuzatilgan. Taxlillar ularni $GaAs_{1-\delta}Bi_\delta$ turidagi birikmaga tegishli ekanligini tasdiqlagan. Mazkur (311) va (511) tuzilmaviy akslanishlarnin eksperimental natijalaridan (3.4.1) ifoda yordamida $GaAs_{1-\delta}Bi_\delta$ kristall panjarasining doimiysini 0.5941 nm ga tengligi aniqlandi. Qatlam asos kristall

panjarasi va $GaAs_{1-\delta}Bi_\delta$ birikma kristall panjarasining nomutonositligi quyidagi ifoda yordamida aniqlandi.

$$\xi = 2|a_{M.c} - a_{GaAsBi}| / (a_{M.c} + a_{GaAsBi}), \quad (4)$$

bizning xol uchchun y 0.048 ga teng, ya'ni (100) $GaAsBi/ZnSe$ tizim nomutonositligi 4.8% ni tashkil qiladi. Bunday farq o'z navbatida Plyonkaning sirtiy sohalarida nanoobyektlarning shakllanishiga olib keladi [98]. Bundan tashqari [117] tadqiqot ishi mualliflari nanokirishmalar hamda asosiy qatlam materiallari atomlarining kovalent bog'lanish energiyasiga asoslanib, bunday nanoobyektlarni "kvant nuqtalar" va "kvant quduqlar"ga quyidagicha ajratishni taklif qilishadi: nanokirishmaning man etilgan energetik sohasi ($E_{g,A}$) asosiy yarimo'tkazgichli qatlam sohasidan ($E_{g,B}$) katta bo'lganida ($E_{g,A} > E_{g,B}$) kvant nuqta shakllanadi, aksincha bo'lgan holatlarda esa ($E_{g,A} < E_{g,B}$) keyin kvant o'ralari hosil bo'ladi. Shuningdek, qattiq qorishma tashkillovchilarining kimyoviy tarkiblaridan $GaAs_{1-\delta}Bi_\delta$ man etilgan sohasi energiya qiymatini quyidagi ifoda yordamida hisoblab topildi [10]:

$$E_{GaAsBi} = (1-\delta)E_{g,GaAs} + (\delta)E_{g,GaBi} - \xi(\delta)(1-\delta) \quad (5)$$

Bu yerda δ – rentgenospektral mikrotahlil natijalari asosida aniqlangan $GaAs_{1-\delta}Bi_\delta$ birikmasidagi As va Bi atomlarining miqdori va uning qiymati $\delta=0.135$. ξ - (4) ifoda yordamida aniqlangan GaAs va GaBi birikmalari kristall panjaralarining nomutonositlik va bizning hol uchun 0.18 ni qabul qiladi. Hisoblashlar asosida $GaAs_{1-\delta}Bi_\delta$ man etilgan sohasi energiya qiymatini ~ 1.3 eV ga tengligi aniqlandi. Bu qiymat epitaksial qatlam asos man etilgan soha energiyasining ($E_{M.c} = 1.53$ eV) eksperimental qiymatidan kichik. Bu esa o'z navbatida $GaAs_{1-\delta}Bi_\delta$ birikmasi nanokirishmalarni, ya'ni Plyonka sirtiy sohalarida kvant o'ralarini o'z-o'zidan shakllantiriyotganidan dalolat beradi [11]. Bunday kvant o'ralarining o'lchamlarini (311) va (511) tuzilmaviy akslanishlarning eksperimental qiymatlari asosida (2) formula yordamida aniqlanganda 30.6 va 56.2 nmlarni tashkil qildi. 10^5 va 10^6 gamma nurlanishlari ta'sirida $GaAs_{1-\delta}Bi_\delta$ kvant o'ralarning o'lchamari deyarli o'zgramagan. Shunday qilib, o'tirilgan plyonkalarda shakllangan Ge nanoobyektlari va $GaAs_{1-\delta}Bi_\delta$ kvant o'ralari

energetik jihatdan barqaror holatida bo'lib, kovalent bog'lanishlar bilan muskamlanganligi tufayli 10^5 va 10^6 gamma nurlanishlar ularning o'lchamlarini o'lchamlari o'zgartirish uchun utarli emasligi aniqlandi.

XULOSA

Shunday qilib, o'stirilgan epitaksial plyonkalar o'zida tashkillovchilarining molyar miqdori ($0 \leq x \leq 0.725$ va $0 \leq y \leq 0.638$) asta-sekin o'zgarib boradigan, o'rin almashinuvchi $(\text{ZnSe})_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$ qattiq qorishmalarini namoyon qilishi aniqlandi. (400) tuzilmaviy akslanishning yarimkengligi ($\text{FWHM} = 4.5 \cdot 10^{-3}$ rad) va yuqori intensivligi ($1,2(10^5 \text{ imp}(\text{sek}^{-1}))$ egaligi o'stirilgan plyonkalar kristall panjarasining mukammalik darajasi yetarlicha yuqori va sfalerit tuzilishga ega va (100) kristallografik oriyentatsiyaga mansub monokristall ekanligini tasdiqladi. $(\text{ZnSe})_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$ epitaksial qatlami 10^5 radli gamma nurlari bilan nurlantirilganda plyonkaning kristall panjarasida barqarorlashuv yuzaga kelishi va mukammallik darajasi ortishi aniqlandi. 10^6 radli gamma nurlar $(\text{ZnSe})_{1-x-y}(\text{Ge}_2)_x(\text{GaAs}_{1-\delta}\text{Bi}_\delta)_y$ epitaksial qatlamda nuqsonlarni ortishi natijasida subkristallitlarning o'lchamlari kamayishiga, shuningdek, kristall panjaraning mukammallik darajasi kamayishiga olib kelishi aniqlandi. Ge atomlari plyonkaning ZnSe molekularini asosida tashkillangan subkristallarining bo'linish chegeralarida $21 \div 27$ nm o'lchamdagi nanokristallarini shakllantirishi aniqlandi. O'stirilgan plyonkalarining sirtiy sohalarida $\text{GaAs}_{1-\delta}\text{Bi}_\delta$ birikmalari 30.6 va 56.2 nmli kvant o'ralarini o'z-o'zidan shakllantirishi aniqlandi. O'stirilgan plyonkalarda shakllangan Ge nanoobyektlari va $\text{GaAs}_{1-\delta}\text{Bi}_\delta$ kvant o'ralari energetik jihatdan barqaror holatida bo'lib, kovalent bog'lanishlar bilan mustahkamlanganligi tufayli 10^5 va 10^6 gamma nurlanishlar ularning o'lchamlarini o'lchamlari o'zgartirish uchun yetarli emasligi aniqlandi.

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VAKUUMDA VA CuCl_2 BUG'LARI ISHTIROKIDA HAVODA TERMİK ISHLOV BERILGAN CdSe:Cu,Cl PLYONKALARINING MIKROSTRUKTURASINI TADQIQ QILISH

B.K. Tuychibayev, N.X.Yuldashev

Farg'ona davlat texnika universiteti e-mail: uzferfizika@mail.ru

2025-yil 17-avgustda berilgan; 2025-yil 1-noyabrda chop etish uchun qabul qilingan

Annotatsiya. Turli texnologik parametrlar bilan tayyorlangan plyonkalar bo'yicha rentgenografik va elektron-mikroskopik tadqiqotlar natijalari keltirilgan. Taglik harorati 250 dan 400 °C gacha oshganda, tekstura o'qi tarqalish burchagi kattalashadi, geksagonal modifikatsiya ulushi ortadi, kristallitlar o'lchamlari va rentgen nurlari koherent sochilish sohalari kengayadi. Havo muhitida CuCl_2 ishtirokida ($T_{\text{ТВ}} = 300^\circ\text{C}$) qizdirilgandan so'ng, taglik harorati 250 °C bo'lgan plyonkalar kristallitlari $(111)_k + (0002)_r$ tekisligidan, taglikga parallel bo'lgan $(311)_k + (1\bar{1}\bar{2})_r$ tekisligi orqali $(10\bar{1}3)$ tekisligiga qayta yo'naltiriladi.

Kalit so'zlar: fotosezgir plyonka, CdSe:Cu,Cl , bo'ylama fotoo'tkazuvchanlik, polikristall, tekstura, taglik harorati, termik ishlov berish, sezgirlik kinetikasi, rentgen nurlarining koherent sochilish sohasi o'lchami

Аннотация. Приведены результаты рентгенографических и электронно-микроскопических исследований пленок с различными технологическими параметрами. Показано, что с ростом температуры подложки от 250 до 400 °C увеличивается угол рассеяния оси текстуры, возрастает доля гексагональной модификации, размеры кристаллитов и области когерентного рассеяния рентгеновских лучей. После отжига на воздухе в присутствии CuCl_2 ($T_{\text{ТВ}} = 300^\circ\text{C}$) в пленках, полученных при $T_t = 250^\circ\text{C}$, происходит переориентация кристаллитов от плоскости $(111)_k + (0002)_r$, параллельной плоскости подложки, к $(10\bar{1}3)$ через плоскость $(311)_k + (1\bar{1}\bar{2})_r$.

Ключевые слова: фоточувствительная пленка, CdSe:Cu,Cl продольная фотопроводимость, поликристалл, текстура, температура подложки, термическая обработка, кинетика очувствления, размер когерентного рассеяния рентгеновских лучей.

Abstract. The results of X-ray diffraction and electron-microscopic studies of films prepared under various technological conditions are presented. It is shown that as the substrate temperature increases from 250 to 400 °C, the texture axis scattering angle increases, the fraction of the hexagonal phase grows, and both the crystallite size and the coherent X-ray scattering region expand. After annealing in air in the presence of CuCl_2 ($T_{\text{ann}} = 300^\circ\text{C}$), films deposited at a substrate temperature of 250 °C undergo reorientation of crystallites from the $(111)_k + (0002)_r$ plane, which is parallel to the substrate, toward the plane through the plane.

Keywords: photosensitive film, CdSe:Cu,Cl , longitudinal photoconductivity, polycrystal, texture, substrate temperature, thermal treatment, sensitization kinetics, coherent X-ray scattering domain size.

1.Kirish. Kadmiy selenid polikristall plyonkalari optoelektronika va fotonika sohalarida keng qo'llaniladi, xususan, xotira elementlari, ko'rinadigan va γ -nurlanish detektorlar, yorug'lik diodlari va fotosezgir rezistorlar sifatida [1-5]. Plyonkalar asosida ishlaydigan fotoelektrik qurilmalarning samaradorligini belgilovchi turli parametrlar - fotootkazuvchanlik, tashuvchilar harakatchanligi va ularning yashash davri, fotosezgirlik, lyuks-amper koeffitsienti va boshqa xususiyatlar faol o'rganilmoqda. Shunga qaramay, CdSe polikristall plyonkalarining substruktur parametrlarini rentgen va mikrofotografik usullar yordamida aniq miqdoriy o'rganishlar kam amalga oshirilgan. Ushbu ishning maqsadi - CdSe:Cu,Cl fotosezgir plyonkalarining yuzasi morfologiyasini o'rganish va ularning substrukturini, taglik harorati va toza vakuum hamda CuCl_2 bug'lari mavjud bo'lgan havo muhitida termik ishlov berish vaqti kabi fizik-texnologik omillarga bog'liq holda tahlil

qilishdir. Bundan tashqari, turli texnologik parametrlar bilan tayyorlangan plyonkalar bo'yicha elektronografik va elektron-mikroskopik tadqiqotlar ham o'tkazilgan.

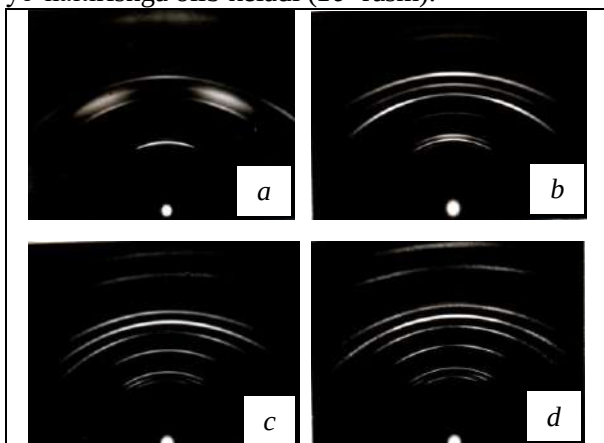
Ma'lumki, kadmiy selenidi kristallanish jarayonida asosiy kristall shakli to'g'ri tetraedr bo'lib, uning to'rt burchagida boshqa element atomlari joylashadi. Shu kabi tetraedrlardan ikki turdagi tuzilmalar hosil bo'lishi mumkin: sfalerit (kubik) va vurtsit (geksagonal) [6-9]. Ikkala tuzilma holatida ham ikkinchi koordinatsion sfera ichidagi har bir atom 12 ta o'sha element atomlari bilan o'ralgan. Umumiy tuzilmalar orasidagi farq uchinchi koordinatsion sferadagi atomlarning joylashishida namoyon bo'ladi, chunki vurtsit tuzilmasidagi uchinchi koordinatsion sferadagi atomlarga bo'lgan masofa sfaleritga nisbatan biroz kichikroq [10,11]. Shuning uchun, olish sharoitlariga qarab, plyonkalar ikkala modifikatsiyada ham bo'lishi mumkin.

Ushbu ishda olib borilgan rentgen tadqiqotlari shuni ko'rsatdiki, CdSe boshlang'ich materialini vakuumda ($\sim 10^{-5}$ mm simob ustuni bosimida) bug'latish orqali, bug' oqimi shisha tagliklarga perpendikulyar tushirilgan holda, kondensatsiya tezligi 2500 Å/min bilan hosil qilingan va qalinligi 5–10 mkm bo'lgan plyonkalar, $T_t=250-400^\circ\text{C}$ oralig'ida teksturalangan va ko'p fazali ekanligi aniqlangan. Ular kubik va geksagonal modifikatsiyalardan tashkil topgan.

2. CuCl_2 mavjudligida havoda va vakuumda fotosensibilizatsiya qilingan plyonkalarni substrukturasi. Natijalarni muhokama qilish.

CdSe yangi tayyorlangan plyonkalarining fotosezgirlikni oshirishning eng samarali usullaridan biri - ularni CuCl_2 ishtirokida havoda termik ishlov berishdir (TIB) [12, 13]. Bu yerda CuCl_2 muhitida TIBdan keyin CdSe plyonkalarining substrukturasi parametrlarini o'lchash, shuningdek, [14–16] ishlarida taklif qilingan usulga muvofiq, toza vakuumda optimal tarzda fotosezgirlikni oshirilgan plyonkalar ko'rib chiqiladi. Eksperimental tadqiqotlar shuni ko'rsatdiki, eng barqaror va parametrlarini takrorlash mumkin bo'lgan fotosezgir plyonkalarni CuCl_2 ishtirokida havoda 300°C temperaturada 4–6 minut davomida termik ishlov berish orqali olish mumkin.

Aniqlanishicha, $T_t=250^\circ\text{C}$ da olingan plyonkalarni qisqa muddatli TIB dan keyin, tekstura o'qi sochilish burchagi va $(311)_k + (11\bar{1}2)_r$ chizig'ining intensivligi ortadi (1a,b-rasm) ($t=2$ min). TIB vaqtini yanada oshirish kristallitlarning tekisligining taglik tekisligiga parallel tarzda qayta yo'naltirishga olib keladi (1c- rasm).



1-rasm. CuCl_2 ishtirokida havoda TIB o'tkazilgan CdSe plyonkalarining $T_t=250^\circ\text{C}$ da olingan holatdagi rentgenogrammalari: a – boshlang'ich holat; b – $t = 2$ min.; c – $t = 3$ min.; d – $t = 4$ min.

o'lchamlarini oshiradi, ammo ularning asosiy orientatsiyasida sezilarli o'zgarish kuzatilmaydi (1d-rasm). Shu bilan birga, shuni ta'kidlash kerakki, bu holatda tartibsiz yo'naltirilgan kristallitlar ham mavjud.

Yuqoriroq T_t da olingan plyonkalarda qisqa muddatli TIB ($t=1$ min) kristallitlarning tekisligi taglikga parallel bo'lgan $(311)_k + (11\bar{1}2)_r$ tekisligi bo'yicha ustun yo'nalishga ega bo'lishiga olib keladi. Biroq TIB vaqtining ortishi bilan yana $(111)_k + (0002)_r$ yo'nalishi ustunlik qila boshlaydi. Bu holatning sababi shundaki, bu sharoitda TIB temperaturasi plyonka bug'latilgan temperaturaga teng yoki undan past bo'ladi. Kattaroq TIB vaqtlarida kristallitlar o'lchamining ortishi Cu ning plyonka hajmiga diffuziyanishi bilan bog'liq. Ma'lumki, mis (Cu) A_2B_6 tipidagi birikmalarda rekristallanishni faollashtiruvchi element hisoblanadi, shuning uchun nisbatan past TIB temperaturasida ham kristallitlar o'lchami ortadi. Kogerent sochilish sohalarining o'lchamini aniqlash bo'yicha eksperimental natijalar ham TIB dan so'ng ularning ortishini ko'rsatadi. $T_t = 350^\circ\text{C}$ da olingan plyonkalar – ya'ni fotoo'tkazuvchanlikning bo'ylama rejimi eng yaxshi namoyon bo'ladigan optimal taglik temperaturasi – 6 minut davom etgan TIBdan so'ng $D_{\text{KSS}}=120$ nm ga yetadi, dislokatsiya zichligining minimal qiymati esa $2 \cdot 10^{10} \text{ cm}^{-2}$ ga teng bo'ladi.

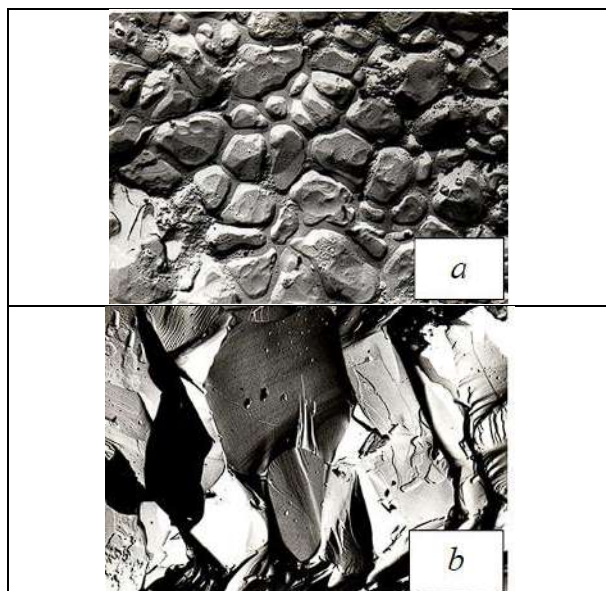
Eksperimental natijalar asosida aniqlanishicha, (333) chizig'i bo'yicha hisoblangan kristall panjarasining parametri TIB vaqti ortishi bilan 0,6082 nm dan 0,60817 nm gacha kamayadi.

2a-rasmda 4 minut davomida termik ishlov berilgan ($T_t = 350^\circ\text{C}$) plyonkalar yuzasining elektron mikrofotosuratlar keltirilgan. Ko'rinib turibdiki, kichik o'lchamli kristallitlar ham mavjud bo'lsa-da, umuman olganda, shu T_t da olingan yangi tayyorlangan plyonkalarga nisbatan kristallitlarning o'rtacha o'lchamlari oshgani kuzatiladi.

Plyonkalarining ko'ndalang sindirilgan qismi bo'yicha olingan elektron mikrofotosuratlar ularning ustuncha strukturasini asosan plyonkaning butun qalinligi bo'ylab tarqalganini ko'rsatadi (2b-rasm).

Toza vakuumda o'tkazilgan TIB holatida plyonkalarining optimal qizdirish harorati 480°C ni tashkil etadi. 550°C dan yuqori haroratlarda plyonkaning taglik sirtidan sublimatsiyasi kuzatiladi, 450°C dan past haroratlarda esa yuqori fotosezgirlikka erishish uchun yanada uzoqroq qizdirish vaqti talab etiladi.

Uzoqroq TIB plyonkalar kristallitlarining



2-rasm. CdSe plyonkalarining CuCl₂ ishtirokida havoda termik ishlov berilgandan keyingi ($t=4$ minut, $T_t = 350^\circ\text{C}$) yuzasining elektron mikrofraktogrammalari (a) va ko'ndalang sinig'ining (b) elektron mikrofraktogrammalari.

Vakuumda optimal darajada sezgirligi oshirilgan CdSe plyonkalarining tuzilishini o'rganish shuni ko'rsatadiki, bu holda yangi tayyorlangan plyonkalarga nisbatan kristallitlar o'lchamining ortishi kuzatiladi.

Kristallitlarning ustun yo'nalishi $(111)_k + (0002)_r$ bo'lib, $(10\bar{1}3)$ chiziq intensivligi ortadi.

Plyonka yuzasidan elektronlarning qaytishi natijasida olingan elektrondogrammada ham boshqa teksturalar kuzatiladi. Biroq bu holda tartibsiz yo'nalgan kristallitlar soni ko'proq. Geksagonal modifikatsiya ulushi 67 % ni tashkil qiladi. $T_t = 350^\circ\text{C}$ da olingan plyonkalar uchun rentgen nurlarining kogerent sochilish sohalari o'lchami (D_{kss}) 97 nm dan 180 nm gacha oshadi, $T_t = 400^\circ\text{C}$ da olingan plyonkalar uchun esa 200 nm gacha yetadi.

Dislokatsiyalar zichligining minimal qiymati mos ravishda $3,2 \cdot 10^{10}$ dan $9,2 \cdot 10^9$, $7,5 \cdot 10^9 \text{ cm}^{-2}$ gacha kamayadi. Elektron mikroskopiya usullari yordamida taglik tekisligiga parallel joylashgan plyonka yuzasining morfologiyasini o'rganish natijalariga ko'ra, o'rtacha kristallit o'lchamlari 2,25 mkm gacha oshadi (2a-rasm). Ayrim kristallitlarning o'lchami esa 3–4 mkm gacha yetadi. Taglikka nisbatan normal yo'nalishda plyonkalarining ko'ndalang tuzilishi 2b-rasmda keltirilgan fraktogrammaga ko'ra monolit ko'rinishga ega bo'ladi.

Xulosa qilib aytganda, CuCl₂ ishtirokida ($T_{TIB}=300^\circ\text{C}$) havoda TIB o'tkazilganda, $T_t=250^\circ\text{C}$ da olingan plyonkalarda kristallitlarning $(111)_k + (0002)_r$ tekisligi taglikka parallel yo'nalishdan

$(311)_k + (1\bar{1}\bar{1}2)_r$ tekisligi orqali $(10\bar{1}3)$ tekisligiga qayta yo'nalishi kuzatiladi. Yuqoriroq T_t da olingan plyonkalarda ham TIB vaqtining ortishi kristallitlarning taglikka parallel $(311)_k + (1\bar{1}\bar{1}2)_r$ tekisligi bo'yicha ustun yo'nalishiga olib keladi, biroq TO vaqtining yanada oshishi bilan yana $(111)_k + (0002)_r$ yo'nalishi ustunlik qila boshlaydi. Bunda kristallitlar o'lchamining kattalashuvi, kogerent sochilish sohasi D_{kss} ning ortishi, shuningdek, panjara parametri a ning kamayishi va dislokatsiyalarning minimal zichligi ρ_D ning pasayishi kuzatiladi. Vakuumda uzoq muddatli TIB ($t=120$ min) kristallitlar o'lchamining va D_{kss} ning yanada sezilarli oshishiga olib keladi, plyonkalar ko'ndalang yo'nalishda monoblok tuzilishga ega bo'ladi.

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Mualliflar haqida ma'lumot

Yoldashev Nosirjon Haydarovich – fizika-matematika fanlari doktori, Farg'ona davlat texnika universiteti, "Fizika" kafedrası professori.

Tuychibaev Bahodir Kurbonaliyevich – Farg'ona davlat texnika universiteti, "Fizika" kafedrası assistenti.
Telefon: +99893-971-74-76

Qisman to'yingan g'ovak-elastik muhitda Reley to'liqlari

Matanov Muhammad Charshamiyevich

Qarshi davlat universiteti

E-mail: mm939011166@mail.ru

2025-yil 2-iyulda berilgan; 2025-yil 28-oktyabrda chop etish uchun qabul qilingan

Annotatsiya: Qisman to'yingan g'ovak muhitda Reley to'liqlari tarqalishi o'rganildi. Muhit qattiq jism va aralashmaydigan ikki xil yopishqoq suyuqlik bilan to'ldirilgan bo'shliqlar tizimi sifatida qaraldi. Model qisman suyuqlik va gaz pufakchalarini ham qamrab oldi. Izotrop muhitda potensial funksiyalar uchta hajmiy va bitta siljish to'liqini hosil qiladi hamda chuqurlikka qarab eksponensial so'nadi. Dissipativ xususiyat tufayli Reley to'liqi chuqurlikka qarab so'nib, sirt kuchlanishlarini nolga keltiradi.

Ularning mavjudligi sekulyar irratsional tenglama orqali ifodalanib, polinom shaklida ildizlari topildi. Shu asosda Reley to'liqlarining tarqalishi va xususiyatlari tahlil qilindi. Tezlik va amplituda muhitdagi o'rtacha siljish trayektoriyasini aniqlashda qo'llandi. To'liqlarning mavjudligi sekulyar tenglamadagi parametrlar qiymatlariga bog'liq bo'lib, sonli modellastirish orqali to'yinish darajalari belgilandi. Shuningdek, tezlik, sifat faktori, polarizatsiya chastotasi, kapillyar bosim, yopishqoqlik va jism anelastikligining ta'siri o'rganildi.

Kalit so'zlar: yuzaki to'liqlar, erkin tebranishlar; seysmik so'nish, nazariy seysmologiya, to'liqlarning tarqalishi.

Аннотация: В частично насыщенной пористой среде исследовано распространение волн Рэлея. Среда рассматривалась как твёрдое тело с системой пустот, заполненных двумя несмешивающимися вязкими жидкостями. Модель также учитывает случай частичного насыщения жидкостью и наличие газовых пузырьков. В изотропной среде потенциальные функции формируют три объёмные и одну сдвиговую волну, которые экспоненциально затухают с глубиной. Вследствие диссипативных свойств волна Рэлея затухает вглубь и приводит поверхностные напряжения к нулю.

Их существование выражается через секулярное иррациональное уравнение, приведённое к полиномиальной форме, корни которого были найдены. На этой основе проведён анализ существования и характеристик волн Рэлея. Скорость и амплитуда использовались для определения средней траектории смещений в среде. Существование волн зависит от параметров секулярного уравнения; с помощью численного моделирования определены уровни насыщения, при которых они возникают. Кроме того, изучено влияние скорости, фактора качества, частоты поляризации, капиллярного давления, вязкости жидкости и аanelastичности твёрдого тела на изменение волн Рэлея.

Ключевые слова: поверхностные волны, свободные колебания, сейсмическое затухание, теоретическая сейсмология, распространение волн.

Abstract: The propagation of Rayleigh waves in a partially saturated porous medium has been investigated. The medium was modeled as a solid body with a system of pores filled with two immiscible viscous fluids. The model also covers the case of partial liquid saturation with gas bubbles. In an isotropic medium, potential functions generate three bulk waves and one shear wave, which decay exponentially with depth. Due to dissipative properties, the Rayleigh wave attenuates with depth and reduces surface stresses to zero.

Their existence is expressed by a secular irrational equation, reduced to a polynomial form whose roots were determined. Based on this, the existence and propagation characteristics of Rayleigh waves were analyzed. The velocity and amplitude were applied to determine the average displacement trajectory in the medium. The existence of the waves depends on the parameter values of the secular equation; numerical modeling was used to define the saturation levels at which they may arise. Furthermore, the influence of velocity, quality factor, polarization frequency, capillary pressure, fluid viscosity, and solid anelasticity on Rayleigh wave variations was examined.

Keywords: surface waves, free oscillations, seismic attenuation, theoretical seismology, wave propagation.

I. Kirish

Yer qobig'ining ko'plab magmatik, metamorfik va cho'kindi jinslari g'ovak tuzilishga ega bo'lib, ular bir yoki bir nechta suyuqliklar bilan to'yingan bo'lishi mumkin. Bunday hollarda hosil bo'ladigan qisman

to'yingan g'ovak-elastik muhit akustika, qurilish injiniringi, seysmologiya va yer osti resurslarini o'rganishda katta ahamiyatga ega.

Biot tomonidan ishlab chiqilgan tenglamalar g'ovak-elastik muhitda to'liq tarqalishini ifodalashda asosiy nazariy

poydevor bo'lib xizmat qiladi. Keyinchalik Berryman, Pride hamda boshqa tadqiqotchilar uni ko'p fazali suyuqliklar uchun kengaytirgan. Aralashma nazariyasi asosida qisman to'yinmagan muhitlarda qo'shimcha siqiluvchan to'lqinlarning mavjudligi ham nazariy jihatdan asoslab berilgan.

So'nggi yondashuvlarda kapillyar bosim, yopishqoqlik va inersiya bog'lanishi hisobga olingan murakkab matematik modellar ishlab chiqilgan. Reley to'lqinlarining tarqalishi esa murakkab irratsional sekulyar tenglamalar orqali tavsiflanib, ularni polinom shakliga keltirish va sonli usullar yordamida yechish zarurati tug'iladi. Shu asosda, ushbu tadqiqotda ikki aralashmaydigan yopishqoq suyuqlik bilan qisman to'yingan g'ovak-elastik muhitda Reley to'lqinlarining tarqalishi o'rganiladi. Modellash orqali to'lqinlarning tezligi, sifat faktori, polyarizatsiyasi hamda qisman to'yinmaganlik, suyuqlik yopishqoqligi va jismning anelastikligi ta'siri tahlil qilinadi.

I. Asosiy tenglamalar

Kompozit g'ovak muhit uchta tarkibiy qismlardan tashkil topgan: qattiq jism, g'ovak-gaz va g'ovak-suyuqlik. Ular mos ravishda "s", "g" va "l" indeksleri bilan belgilanadi. Umumiy bog'langan g'ovaklilik (f) ga ega g'ovak agregatda hajmiy ulushlar quyidagicha aniqlanadi:

$$\delta_s = 1 - f, \delta_g = (1 - \sigma)f, \delta_l = \sigma f, \quad (1)$$

bu yerda σ – bog'langan g'ovak fazodagi suyuqlik to'yinishining ulushi.

Izotrop g'ovak jismlarda tashqi kuchlar bo'lmasa tarkibiy zarrachalarning past chastotali tebranishlari uchun harakat tenglamalari quyidagicha ifodalanadi:

$$\begin{aligned} \delta_s \tau_{ij,j}^{(p)} &= \delta_s \rho_s \ddot{u}_i - q_g (\ddot{v}_i - \ddot{u}_i) - q_l (\ddot{w}_i - \ddot{u}_i), \\ -\delta_g p_i^{(g)} &= \delta_g \rho_g \ddot{v}_i + q_g (\ddot{v}_i - \ddot{u}_i), \\ -\delta_l p_i^{(l)} &= \delta_l \rho_l \ddot{w}_i + q_l (\ddot{w}_i - \ddot{u}_i), \end{aligned} \quad (2)$$

bu yerda $p^{(g)}, p^{(l)}$ – gaz va suyuqlik fazalaridagi bosimlar, $\tau_{ij}^{(p)}$ esa drenajlangan g'ovak muhitdagi kuchlanish tenzori. ρ – tarkibiy qismlarning solishtirma zichliklari. u_i, v_i, w_i mos ravishda qattiq, gaz va suyuqlik

zarrachalarining siljish komponentlarini bildiradi. Indeksler $i = 1, 2, 3$ qiymatlar qabul qilishi mumkin. Indeksning takrorlanishi yig'indini anglatadi. Nuqta – vaqt bo'yicha xususiy hosila, indeksdan oldingi vergul esa fazodagi xususiy hosilani bildiradi.

Gaz (q_g) va suyuqlik (q_l) uchun dissipasiyalash koeffitsiyentlari quyidagicha aniqlanadi:

$$q_k = \eta_k \delta_k^2 / (\chi \chi_k), (k = g, l), \quad (3)$$

bu yerda η_k va χ_k mos ravishda yopishqoqlik va suyuqlik fazasining nisbiy o'tkazuvchanligini bildiradi. χ esa g'ovak muhitning ichki o'tkazuvchanligi.

G'ovak muhitdagi kuchlanishlar va muhitdagi bosimlar quyidagicha beriladi:

$$\begin{aligned} \delta_s \tau_{ij}^{(p)} &= (a_{11} \nabla \cdot u + a_{12} \nabla \cdot v + a_{13} \nabla \cdot w) \delta_{ij} + \\ &+ G_p (u_{i,j} + u_{j,i} - \frac{2}{3} u_{k,k} \delta_{ij}), \end{aligned} \quad (4)$$

$$-\delta_g p^{(g)} = (a_{21} \nabla \cdot u + a_{22} \nabla \cdot v + a_{23} \nabla \cdot w), \quad (5)$$

$$-\delta_l p^{(l)} = (a_{31} \nabla \cdot u + a_{32} \nabla \cdot v + a_{33} \nabla \cdot w), \quad (6)$$

Bu yerda δ_{ij} — Kroneker hadi. Koeffitsiyentlar a_{ij} muhitning elastik doimiyliklarini bildiradi. O'lchanadigan kattaliklar orqali ular quyidagicha aniqlanadi:

$$\begin{aligned} a_{11} &= K_p, a_{12} = a_{21} = K_g \delta_s (1 - \sigma) (K_c + K_l) / K, \\ a_{13} &= a_{31} = K_g \delta_s \sigma (K_c + K_g) / K, \end{aligned}$$

$$a_{22} = K_g \delta_g [(1 - \sigma) K_l + K_c] / K,$$

$$a_{23} = a_{32} = K_g K_l (1 - \sigma) \delta_l / K,$$

$$a_{33} = K_l \delta_l (K_g \sigma + K_c) / K,$$

$$K = K_g \sigma + K_c + K_l (1 - \sigma).$$

Bu yerda: K_p, K_g, K_l — mos ravishda g'ovak muhitning qattiq, gaz va suyuqlik fazalarining hajmiy modullari, G_p — g'ovak muhitning qattiqligi, $\sigma \in (0, 1)$ — g'ovak jismlarning to'yinish darajasi ($\sigma = 1$ to'liq to'yinganlikni bildiradi), $K_c = \sigma K_{cap}$, bu yerda K_{cap} namlanuvchi va namlanmaydigan

suyuqliklar orasidagi makroskopik kapillyar bosimga mos keluvchi ekvivalent hajmiy modul.

$$\delta_s \rho_s \ddot{u} = \nabla[(a_{11} + \frac{1}{3}G_p)\nabla \cdot u + a_{12}\nabla \cdot v +$$

$$+ a_{13}\nabla \cdot w] + G_p \nabla^2 u + q_g(\dot{v} - \dot{u}) + q_l(\dot{w} - \dot{u}),$$

$$\delta_g \rho_g \ddot{v} = \nabla(a_{21}\nabla \cdot u + a_{22}\nabla \cdot v + a_{23}\nabla \cdot w) -$$

$$- q_g(\dot{v} - \dot{u}),$$

$$\delta_l \rho_l \ddot{w} = \nabla(a_{31}\nabla \cdot u + a_{32}\nabla \cdot v + a_{33}\nabla \cdot w) -$$

3. Umumiy yechim: garmonik to'liqlar

Vektorning odatiy Gelmgolts taqsimoti orqali uchta gomogen izotrop fazadagi siljish vektorlari quyidagicha yoziladi:

$$u = \nabla \phi + \nabla \times S, \quad \nabla \cdot S = 0; \quad (10)$$

$$v = \nabla \psi + \nabla \times G, \quad \nabla \cdot G = 0; \quad (11)$$

$$w = \nabla \xi + \nabla \times L, \quad \nabla \cdot L = 0. \quad (12)$$

Bu yerda ϕ, ψ, ξ — siljish potentsiallari, S, G, L esa solenoidal qismlarni bildiradi. (7)–(9) tenglamalar siljish potentsiallari orqali quyidagi ko'rinishga ega bo'ladi:

$$\delta_s \rho_s \ddot{\phi} = [\tilde{a}_{11} \nabla^2 \phi + a_{12} \nabla^2 \psi + a_{13} \nabla^2 \xi +$$

$$+ q_g(\dot{\psi} - \dot{\phi}) + q_l(\dot{\xi} - \dot{\phi})], \quad (13)$$

$$\tilde{a}_{11} = a_{11} + \frac{4}{3}G_p,$$

$$\delta_g \rho_g \ddot{\psi} = a_{12} \nabla^2 \phi + a_{22} \nabla^2 \psi +$$

$$+ a_{23} \nabla^2 \xi + q_g(\dot{\phi} - \dot{\psi}), \quad (14)$$

$$\delta_l \rho_l \ddot{\xi} = a_{13} \nabla^2 \phi + a_{23} \nabla^2 \psi +$$

$$+ a_{33} \nabla^2 \xi + q_g(\dot{\phi} - \dot{\xi}), \quad (15)$$

$$\delta_s \rho_s \ddot{S} = G_p \nabla^2 S + q_g(\dot{G} - \dot{S}) + q_l(\dot{L} - \dot{S}), \quad (16)$$

$$\delta_g \rho_g \ddot{G} = q_g(\dot{S} - \dot{G}), \delta_l \rho_l \ddot{L} = q_l(\dot{S} - \dot{L}). \quad (17)$$

Agar vaqt bo'yicha garmonik potentsiallar ($\sim e^{-i\omega t}$) qaralsa, ϕ, ψ, ξ lar burchak chastotali garmonik to'liqlarni ifodalaydi. Shunda (13)–(15) tenglamalar quyidagi ko'rinishga keladi:

$$(\tilde{a}_{11} \nabla^2 + \omega^2 \Lambda_s) \phi + (a_{12} \nabla^2 - \omega^2 r_g) \psi +$$

$$+ (a_{13} \nabla^2 - \omega^2 r_l) \xi = 0, \quad (18)$$

$$(a_{12} \nabla^2 - \omega^2 r_g) \phi + (a_{22} \nabla^2 + \omega^2 \Lambda_g) \psi +$$

$$+ (a_{23} \nabla^2) \xi = 0, \quad (19)$$

$$(a_{13} \nabla^2 - \omega^2 r_l) \phi + (a_{23} \nabla^2) \psi +$$

$$+ (a_{33} \nabla^2 + \omega^2 \Lambda_l) \xi = 0, \quad (20)$$

$$\text{bu yerda: } r_g = \frac{1}{\omega} q_g, \quad r_l = \frac{1}{\omega} q_l,$$

$$\Lambda_s = \delta_s \rho_s + r_g + r_l, \quad \Lambda_g = \delta_g \rho_g + r_g, \quad \Lambda_l = \delta_l \rho_l + r_l$$

(19) va (20) tenglamalar quyidagi ikki munosabatga keltiriladi:

$$(A_1 \nabla^4 + \omega^2 B_1 \nabla^2 + \omega^4 C_1) \psi =$$

$$= (A_2 \nabla^4 + \omega^2 B_2 \nabla^2 + \omega^4 C_2) \phi,$$

$$(A_1 \nabla^4 + \omega^2 B_1 \nabla^2 + \omega^4 C_1) \xi =$$

$$(A_3 \nabla^4 + \omega^2 B_3 \nabla^2 + \omega^4 C_3) \phi, \quad (22)$$

Bu munosabatlarni (18) tenglamaga qo'llash orqali quyidagi natija olinadi:

$$[A \nabla^6 + \omega^2 B \nabla^4 + \omega^4 C \nabla^2 + \omega^6 D] \phi = 0. \quad (23)$$

Bu yerda

$$A = \tilde{a}_{11} A_1 + a_{12} A_2 + a_{13} A_3,$$

$$B = \tilde{a}_{11} B_1 + a_{12} B_2 + a_{13} B_3 + \Lambda_s A_1 - r_g A_2 - r_l A_3,$$

$$C = \tilde{a}_{11} C_1 + a_{12} C_2 + a_{13} C_3 + \Lambda_s B_1 - r_g B_2 - r_l B_3,$$

$$D = \Lambda_s C_1 - r_g C_2 - r_l C_3,$$

$$A_1 = a_{22} a_{33} - a_{23} a_{23},$$

$$A_2 = a_{23} a_{13} - a_{12} a_{33},$$

$$A_3 = a_{12} a_{23} - a_{13} a_{22},$$

$$B_1 = a_{22} \Lambda_l + a_{33} \Lambda_g,$$

$$B_2 = a_{33} r_l - a_{12} \Lambda_l - a_{23} r_l,$$

$$B_3 = a_{22} r_l - a_{13} \Lambda_g - a_{23} r_g,$$

$$C_1 = \Lambda_g \Lambda_l, \quad C_2 = \Lambda_l r_g, \quad C_3 = \Lambda_g r_l.$$

(23) differensial tenglama uchta Gelmgolts tenglamasiga ajratiladi:

$$(\nabla^2 + \frac{\omega^2}{\alpha_i^2}) \phi_i = 0, \quad (i = 1, 2, 3). \quad (24)$$

Bu natija uchta P-to'liqlarning mavjudligini bildiradi; ular faza tezliklari ($i = 1, 2, 3$) bilan tarqaladi va mos skalyar potentsiallar ϕ_i bilan aniqlanadi. α_i tezliklar α^2

quyidagi kubik tenglamaning ildizlaridan olinadi:

$$D\alpha^6 - C\alpha^4 + B\alpha^2 - A = 0. \quad (25)$$

Ko'rib chiqilayotgan chiziqli g'ovak muhitda umumiy kengayish potensialining ifodasi quyidagicha yoziladi:

$$\phi = \phi_1 + \phi_2 + \phi_3, \quad (26)$$

shundan (21)–(22) munosabatlarni qo'llash orqali quyidagi ifodalar olinadi:

$$\psi = \mu_1\phi_1 + \mu_2\phi_2 + \mu_3\phi_3, \quad (27)$$

$$\xi = \nu_1\phi_1 + \nu_2\phi_2 + \nu_3\phi_3, \quad (28)$$

bu yerda

$$\mu_j = \frac{A_2 - B_2\alpha_j^2 + C_2\alpha_j^4}{A_1 - B_1\alpha_j^2 + C_1\alpha_j^4},$$

$$\nu_j = \frac{A_3 - B_3\alpha_j^2 + C_3\alpha_j^4}{A_1 - B_1\alpha_j^2 + C_1\alpha_j^4}, \quad (j = 1, 2, 3).$$

Skalyar potensiallarga o'xshash tarzda, vaqt bo'yicha garmonik ($e^{-i\omega t}$) vektor potensiallar ($\mathbf{S}, \mathbf{G}, \mathbf{L}$) uchun (16)–(17) tenglamalar quyidagicha yechiladi:

$$(G_p \nabla^2 + \omega^2 \Lambda_s) \mathbf{S} - \omega^2 r_g \mathbf{G} - \omega^2 r_l \mathbf{L} = 0, \quad (29)$$

$$\{\mathbf{G}, \mathbf{L}\} = \{\mu_4, \nu_4\} \mathbf{S}, \quad \mu_4 = \frac{r_g}{\Lambda_g}, \quad \nu_4 = \frac{r_l}{\Lambda_l}. \quad (30)$$

Yuqoridagi munosabatlardan Gelmgolts tenglamasi olinadi:

$$(\nabla^2 + \frac{\omega^2}{\beta^2}) \mathbf{S} = 0,$$

$$\beta^2 = \frac{G_p}{\Lambda_s - \mu_4 r_g - \nu_4 r_l}, \quad (31)$$

bu esa tezlik β bilan tarqaluvchi kesuvchi to'qlinning mavjudligini belgilaydi.

4. Tekislikda harakat

Dekart koordinatalar sistemasida (x, y, z), g'ovak jism $z > 0$ yarim fazoni egallaydi va $z = 0$ tekislik bilan chegaralangan. Ushbu muhitning izotropiyasi to'qlin harakatini faqat $x - z$ tekislikda o'rganishga imkon beradi, shu sababli barcha kattaliklar y koordinataga bog'liq emas. Shunday qilib, qattiq va suyuqlik

fazalarining tekislikdagi siljishlari quyidagicha ifodalanadi:

$$u_x = \sum_{j=1}^3 \mu_j \frac{\partial \phi_j}{\partial x} + \mu_4 \frac{\partial \phi_4}{\partial z}, \quad (32)$$

$$u_z = \sum_{j=1}^3 \mu_j \frac{\partial \phi_j}{\partial z} - \mu_4 \frac{\partial \phi_4}{\partial x},$$

$$v_x = \sum_{j=1}^3 \mu_j \frac{\partial \phi_j}{\partial x} + \mu_4 \frac{\partial \phi_4}{\partial z}, \quad (33)$$

$$v_z = \sum_{j=1}^3 \mu_j \frac{\partial \phi_j}{\partial z} - \mu_4 \frac{\partial \phi_4}{\partial x},$$

$$w_x = \sum_{j=1}^3 \nu_j \frac{\partial \phi_j}{\partial x} + \nu_4 \frac{\partial \phi_4}{\partial z}, \quad (34)$$

$$w_z = \sum_{j=1}^3 \nu_j \frac{\partial \phi_j}{\partial z} - \nu_4 \frac{\partial \phi_4}{\partial x},$$

Bu yerda $\phi_4 = (-\mathbf{S})$, Siljish potensiallari ϕ_j ($j = 1, 2, 3, 4$) mos ravishda P_1, P_2, P_3 va SV to'qlinlarning tarqalishini bildiradi; ularning tezliklari $\alpha_1, \alpha_2, \alpha_3$ va β ga teng. (4)–(6) tenglamalarda siljish ifodalaridan foydalanish orqali kuchlanish komponentlari va g'ovaklardagi bosimlar potensiallar ko'rinishida olinadi.

Chegaraviy masalani yechishda tegishli shartlar umumiy yechimdan xos ravishda olinadi. Ushbu ishda $x - z$ tekislikdagi zarracha harakati uchun mos chegaraviy shartlar $z = 0$ yuzada qaraladi. Bu sirt erkin sirt deb hisoblanadi, ya'ni unda normal hamda tangensial kuchlanishlar nolga teng bo'lishi kerak. Ochiq g'ovakli sirtlar uchun esa, har bir g'ovak-suyuqlik fazasidagi bosim yuzada nolga teng bo'lishi shart.

5. Reley to'qlinlari

Garmonik tekis to'qlinlar x -yo'nalishida tarqalib, z -yo'nalishda eksponensial so'nadi. Ularning siljish potensiallari quyidagicha tanlanadi:

$$\phi_j = A_j e^{ik(x-ct) - kd_j z}, \quad (j = 1, 2, 3, 4), \quad (35)$$

bu yerda k — gorizontaal to‘lqin soni, c — ko‘rinma faza tezligi va $d_j = \sqrt{1 - \frac{c^2}{\alpha_j^2}}$,

$$(j=1,2,3), \quad d_4 = \sqrt{1 - \frac{c^2}{\beta^2}}.$$

Ushbu potentsiallar (32)–(34) ifodalardan foydalanib, qattiq va suyuq zarrachalarning siljishlarini hisoblash uchun qo‘llaniladi. Keyinchalik (4)–(6) tenglamalar kuchlanishlar va g‘ovak bosimlarini aniqlashda ishlatiladi.

G‘ovak jism sirtining normali z -o‘q bo‘yicha yo‘nalgan tekislik deb qabul qilinadi. Erkin sirt natijaviy kuchsiz sirt deb hisoblanadi. Shunday qilib, $z = 0$ yuzaning har bir nuqtasida natijaviy energiya nolga teng bo‘lishi kerak:

$$\delta_s [\tau_{zz}^{(s)} \dot{u}_z + \tau_{zx}^{(s)} \dot{u}_x] + \delta_g [-p^{(g)}] \dot{v}_z + \delta_l [-p^{(l)}] \dot{w}_z = 0. \quad (36)$$

So‘ngra, chegaraviy shartlar sirtning fizik xususiyatlariga ko‘ra belgilanadi. Agar g‘ovaklar ochiq bo‘lsa, u holda g‘ovak-suyuqlik sirtida bosim kuzatilmaydi. Natijada, agregat kuchlanishning normal va tangensial komponentlari chegarada nol bo‘lishi kerak. Erkin sirt ($z = 0$) uchun mos chegaraviy shartlar quyidagicha yoziladi:

$$\delta_s \tau_{zz}^{(s)} - \delta_g p^{(g)} - \delta_l p^{(l)} = 0 \quad (37)$$

$$\tau_{zx}^{(s)} = 0, \quad p^{(g)} = 0, \quad p^{(l)} = 0$$

Bu shartlar yanada soddalashtirilib quyidagicha yoziladi:

$$\tau_{zz}^{(s)} = \tau_{zx}^{(s)} = p^{(g)} = p^{(l)} = 0.$$

Potentsiallardan (35) hosil qilingan kuchlanish va bosim ifodalari asosida, bu chegaraviy shartlar A_j ($j=1,2,3,4$) uchun to‘rtta chiziqli gomogen tenglamalar sistemasiga aylanadi. Ushbu sistema uchun trivial bo‘lmagan yechim faqat determinant nol bo‘lganda mavjud bo‘ladi, ya‘ni koeffitsiyentlar matritsasi singulyar bo‘lganda. Bu Reley to‘lqinining mavjudligini ta‘minlaydi.

To‘liq to‘yingan g‘ovak muhitda Reley to‘lqinining mavjudligi quyidagi sekulyar tenglama bilan ifodalanadi:

$$4d_4 \prod_{j=1}^3 D_j d_j (2-h) \cdot \left(2 \sum_{j=1}^3 D_j - h \sum_{j=1}^3 D_j X_j \right) = h \frac{c^2}{\beta^2}, \quad (38)$$

bu yerda

$$X_j = \frac{(\tilde{a}_{11} + a_{12} \mu_j + a_{13} \nu_j) b_j}{G_p}, \quad (b_j = \beta^2 / \alpha_j^2),$$

$$Y_j = \frac{(a_{12} + a_{22} \mu_j + a_{23} \nu_j)}{\alpha_j^2},$$

$$Z_j = \frac{(a_{13} + a_{23} \mu_j + a_{33} \nu_j)}{\alpha_j^2},$$

$$D_i = (Y_j Z_k - Y_k Z_j), \quad (i, j, k \text{ sikl}), \quad h = \frac{c^2}{\beta^2}$$

Agar $D_1 = X_1 = 1$ va $D_2 = D_3 = 0$

$D_2 = D_3 = 0$ bo‘lsa, (38) tenglama mukammal elastik qattiq jismdagi klassik Reley to‘lqinining sekulyar tenglamasiga keltiriladi.

Ildizlar mavjud bo‘lganda, (38) tenglama irratsional shaklga ega bo‘ladi va uni oddiy algebraik usullar bilan yechib bo‘lmaydi. Ushbu irratsional tenglama polinom shakliga keltiriladi. Bu jarayon irratsional tenglamada paydo bo‘ladigan kvadrat ildizlarni yo‘qotish uchun kvadratlash va algebraik soddalashtirishlarni o‘z ichiga oladi. Uchtalik kvadratlashdan so‘ng, (38) sekulyar tenglama 16-darajali polinomial tenglamaga keltiriladi. Hosil bo‘lgan algebraik tenglama quyidagicha yoziladi:

$$\sum_{j=0}^{16} g_j h_j = 0, \quad (39)$$

bu yerda g_j koeffitsiyentlar matematik dasturiy ta‘minot (masalan, MATLAB) yordamida quyidagi ifodadan olinadi:

$$(\Gamma_1 - \Delta_1)^4 - 2(\Gamma_1 - \Delta_1)^2 (\Delta_2 d_1^2 d_2^2 + \Gamma_2 d_3^2 d_4^2) + \Delta_4^2 d_1^4 d_2^4 + \Gamma_4^2 d_3^4 d_4^4 = 2\Delta_2 \Gamma_2 d_1^2 d_2^2 d_3^2 d_4^2, \quad (40)$$

bu yerda

$$\begin{aligned}
 d_j^2 &= 1 - b_j h, (j = 1, 2, 3), \quad d_4^2 = 1 - h, \\
 \Delta_1 &= 16d_4^2(D_1^2 d_1^2 + D_2^2 d_2^2), \quad \Delta_2 = 32D_1 D_2 d_1^2 d_2^2, \\
 \Gamma_1 &= s_1^2 h^4 + 2s_1 s_2 h^3 + (s_2^2 + 2s_1 s_3) h^2 + \\
 &\quad + 2s_2 s_3 h + s_3^2 + 16D_3^2 d_3^2 d_4^2, \\
 \Gamma_2 &= -8D_3(s_1 h^2 + s_2 h + s_3), \quad s_1 = \sum_{j=1}^3 D_j X_j, \\
 s_2 &= -2 \sum_{j=1}^3 D_j(1 + X_j), \quad s_3 = 4 \sum_{j=1}^3 D_j.
 \end{aligned}$$

(39) algebraik tenglamaning 16 ta murakkab ildizi mavjud, biroq ular orasida ayrim noto'g'ri ildizlar ham mavjud bo'lib, ular (38) tenglamani qanoatlantirmaydi va shu sababli chiqarib tashlanadi.

5.1. Nomutanosib to'liqlarning tarqalishi

Agar g'ovak suyuqliklari yopishqoq bo'lmasa, muhit nodissipativ bo'lib, haqiqiy sirt to'liqini barcha jism to'liqlaridan sekinroq tarqaladi. Ko'p fazali muhitda, ayniqsa g'ovaklarning biri havo yoki gaz bilan to'yinganda, uchinchi bosim to'liqini tezligi juda kichik bo'lib, sirt to'liqini tezligi qir qilish to'liqini tezligiga nisbatan 0.9 dan 0.1 gacha tushishi mumkin. Bu amalda asoslash qiyin bo'lgani uchun Lo psevdosirt to'liqlari yondashuvini taklif qilgan. Biroq bu usul murakkab sekulyar tenglamaga olib keladi va yechim faqat kompleks ildizlar orqali aniqlanadi. Ushbu tadqiqotda esa murakkab ildizlar dissipativ g'ovak jism sirtida so'nuvchi Reley to'liqlarining mavjudligini isbotlashda qo'llaniladi.

Sirt to'liqining faza tezligi va so'nish koeffitsienti kompleks qiymatlar orqali aniqlanadi. Dissipativ muhitda haqiqiy sirt to'liqlari tez so'nuvchi bo'lib, tekislik bo'ylab tarqaladi va normal yo'nalishda maksimal so'nishga ega bo'ladi. Agar kompleks ildizlar yuzaga kelsa, dissipativ muhitda haqiqiy sirt to'liqlari tarqalmasligi mumkin.

Yopishqoq suyuqlik bilan to'yingan g'ovak-elastik muhit cho'kindi va kollektor jinslari uchun muhim matematik model bo'lib,

unda g'ovaklikdagi yopishqoq yo'qotishlar hisobga olinadi.

Ushbu tadqiqotda, jismning g'ovak elastik ta'siri elastik modullarni kompleks o'zgarishi orqali belgilanadi: $K_p(1 - iQ_b^{-1})$, $G_p(1 - iQ_1^{-1})$, ya'ni hajmiy modul (K_p) va kesuvchi moduli (G_p) kompleks shaklida olinadi. Sifat faktorlari logaritmik dekrement parametrlari (δ_G, δ_E) bilan bog'langan, ular drenajlangan jismning qattiqlik (G) va Yung moduli (E) ga mos keladi.

Ular orasidagi munosabatlar quyidagicha ifodalanadi:

$$Q_r = \frac{\pi}{\delta_G}, \quad \frac{Q_r}{Q_b} \approx 5 \frac{\delta_E}{\delta_G} 4, \quad (41)$$

bu esa jismning so'nishini δ_G va δ_E / δ_G qiymatlari orqali belgilaydi. Reley to'liqlari uchun sifat faktori Q quyidagicha aniqlanadi:

$$Q^{-1} = -\frac{\Im(c^2)}{\Re(c^2)}.$$

$h = h_R + ih_I$ bo'lganda, Reley to'liqlari tarqalishi uchun Q quyidagicha hisoblanadi:

$$Q^{-1} = \frac{h_R - h_I Q_s}{h_R Q_s + h_I}, \quad Q_s^{-1} = -\frac{\Im(\beta^2)}{\Re(\beta^2)}. \quad (42)$$

5.3. Zarralarning harakati

(32)–(34) ifodalar qattiq va suyuqlik zarrachalarining x – z tekislikdagi harakatini belgilaydi. Shundan so'ng, g'ovak muhit uchun o'rtacha siljish komponentlari quyidagicha ifodalanadi:

$$\begin{aligned}
 U &= \delta_s u_x + \delta_g v_x + \delta_l w_x = \\
 &= \sum_{j=1}^3 [(\delta_s + \delta_g \mu_j + \delta_l \nu_j) \frac{\partial \phi_j}{\partial x}] +
 \end{aligned} \quad (43)$$

$$\begin{aligned}
 &+ (\delta_s + \delta_g \mu_4 + \delta_l \nu_4) \frac{\partial \phi_4}{\partial z}, \\
 W &= \delta_s u_z + \delta_g v_z + \delta_l w_z = \\
 &= \sum_{j=1}^3 [(\delta_s + \delta_g \mu_j + \delta_l \nu_j) \frac{\partial \phi_j}{\partial z}] -
 \end{aligned} \quad (44)$$

$$- (\delta_s + \delta_g \mu_4 + \delta_l \nu_4) \frac{\partial \phi_4}{\partial x},$$

Siljish potentsiallari (35) orqali quyidagicha yoziladi:

$$\phi_j = A_j \gamma_j e^{i(kx - \omega t) - kd_j z}, \quad (j = 1, 2, 3, 4), \quad (45)$$

bu yerda $\gamma_j (= A_j / A_1)$ lar (38) sekulyar tenglamaga olib keluvchi chegaraviy shartlarning singulyar sistemasidan aniqlanadi. Ular quyidagicha beriladi:

$$\gamma_2 = Y_3 Z_1 - Y_1 Z_3,$$

$$\gamma_3 = Y_1 Z_2 - Y_2 Z_1,$$

$$\gamma_4 = \frac{2}{h-2} [(Y_2 Z_3 - Y_3 Z_2) d_1 + \gamma_2 d_2 + \gamma_3 d_3].$$

(32)–(34) munosabatlardan foydalanib, oʻrtacha siljish quyidagicha yoziladi:

$$U = i|U_0| e^{i \arg U_0} e^{i \omega (\frac{x}{c} - t)}, \quad (46)$$

$$W = -i|W_0| e^{i \arg W_0} e^{i \omega (\frac{x}{c} - t)},$$

bu yerda

$$U_0 = k A_1 \left[\sum_{j=1}^3 \epsilon_j d_j e^{-k_R z \phi_j} + i \epsilon_4 d_4 e^{-k_R z \phi_4} \right], \quad (47)$$

$$\epsilon_j = (\delta_s + \delta_g \mu_j + \delta_l \nu_j) \gamma_j,$$

$$W_0 = k A_1 \left[\sum_{j=1}^3 \epsilon_j \phi_j e^{-k_R z \phi_j} + i \epsilon_4 e^{-k_R z \phi_4} \right], \quad (48)$$

$$\phi_j = (1 - \frac{c_l}{c_R}) d_j,$$

Bu yerda k_R va k_l mos ravishda kompleks toʻlqinlar soni k ning haqiqiy va mavhum qismlari.

(46) dagi kompleks siljishlarning haqiqiy qismini olsak, gʻovak-suyuqlik muhitning koʻrinma siljishi quyidagicha boʻladi:

$$u = -|U_0| e^{-k_l x} \sin(\arg U_0 + \theta), \quad (49)$$

$$w = -|W_0| e^{-k_l x} \cos(\arg W_0 + \theta).$$

$\theta (= k_R x - \omega t)$ parametr $[0, 2\pi]$ oraliqda oʻzgaruvchi parametr sifatida qaraladi va u $x-z$ tekislikdagi z chuqurlikda zarrachalar harakati konturini belgilaydi.

Boshqacha aytganda, (49) tenglamalar jufti dissipativ gʻovak jismda Reley toʻlqini tarqalishi davomida tebranayotgan zarrachalar chizgan egri chiziqning parametrik tasviridir.

6. Sonli yechim

Gʻovak muhitning fizik modeli sifatida qumtosh qaraladi. Suv va havo pufakchalari bilan toʻlgan qumtosh uchun turli parametr qiymatlari tanlandi. Qumtoshning hajmiy moduli $K_p = 44 \text{ MPa}$, qirqilish moduli $G_p = 26 \text{ MPa}$ va zichligi $\rho_s = 2650 \text{ kg m}^{-3}$ boʻlib, gʻovaklilik $f = 0.5$ deb olinadi.

Gʻovak muhitning hajmiy moduli $K_g = 3.7 \text{ MPa}$ va zichligi $\rho_g = 1.2 \text{ kg} \cdot \text{m}^{-3}$ boʻlgan gaz pufakchalari hamda hajmiy moduli $K_l = 2 \text{ GPa}$, zichligi $\rho_l = 1000 \text{ kg} \cdot \text{m}^{-3}$ boʻlgan suv bilan toʻldirilgan.

Gʻovaklardagi yopishqoq dissipatsiyalar quyidagicha aniqlangan: gaz uchun $q_g = 0.04 \text{ MPa} \cdot \text{s} \cdot \text{m}^{-2}$, suyuqlik uchun $q_l = 1 \text{ MPa} \cdot \text{s} \cdot \text{m}^{-2}$. Doimiy nisbat K_{cap} / K_l orqali K_{cap} qiymati belgilanadi. Toʻlqin chastotasining 1 kHz gacha cheklanishi past chastotali tarqalishni taʼminlaydi.

Stoll (1985) elastik materiallarga yaqin hollarda logaritmik dekrement qiymatlarini $(\delta_G, \delta_E / \delta_G)$ baholagan. Ushbu qiymatlar (41) munosabatlarda qoʻllanilib, gʻovak muhitning elastik modullari uchun sifat faktorlarini hisoblashda ishlatiladi.

6.1. Tezlik va soʻnish

Gʻovak muhitning ushbu sonli modelida Reley toʻlqini ixtiyoriy toʻyinish uchun mavjud boʻlmasligi mumkin ($\sigma \in (0, 1)$). Boshqacha aytganda, tanlangan gʻovak muhitda Reley toʻlqinining mavjudligi uning bogʻlangan gʻovak fazosining toʻyinish darajasiga bogʻliq. Sonli misolning bir qismi Reley toʻlqini mavjud boʻladigan σ (suyuqlik ulushi) diapazonini aniqlashga bagʻishlangan. Boshqa qismida esa zarrachalar harakati Reley toʻlqini mavjud boʻlgan mos ravishda toʻyingan gʻovak muhit uchun hisoblanadi (yaʼni haqiqiy σ).

Mavjud Reley toʻlqini murakkab tezlik qiymati c orqali aniqlanadi, u tezlik nisbati c / β va kesuvchi toʻlqin tezligi β dan

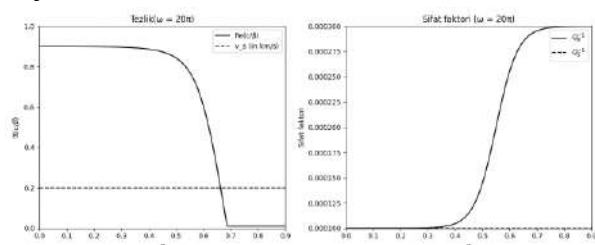
hisoblanadi. Sifat faktori (Q_s) va kesuvchi to'liqning murakkab tezligi Reley to'liqining sifat faktorini (Q^{-1}) hisoblash uchun ishlatiladi.

Keyin modeldagi turli parametrlarning ta'siri Reley to'liqining mavjudligiga (ya'ni σ diapazoniga) va to'liqning qutblanishiga (mos σ uchun) o'rganiladi. Ushbu parametrlar to'liq chastotasi, g'ovaklardagi kapillyar bosim, g'ovak suyuqlikning yopishqoqligi va g'ovak muhitning noelastikligini ifodalaydi.

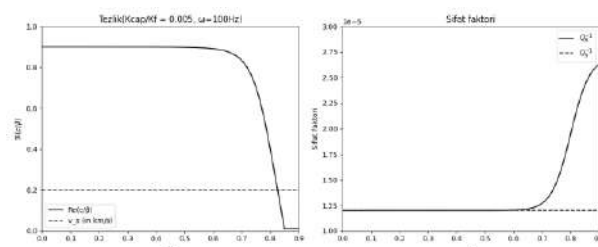
Tanlangan sonli model uchun shuni kuzatish mumkinki, bu xususiyatlardagi o'zgarishlarning hech biri kesuvchi to'liq tezligi (β) va uning sifat faktori (Q_s^{-1}) ga sezilarli ta'sir ko'rsatmaydi.

O'lchamsiz haqiqiy qiymatlar $\Re(c/\beta)$ va Q^{-1} Reley to'liqining tarqalishini o'rganishda muhim miqdordir. Shuni ham qayd etish kerakki, $v_s = \sqrt{\Re(\beta^2)}$, Q_s^{-1} , miqdorlari Reley to'liqning murakkab tezligini c ni hisoblash imkonini beradi $\Re(c/\beta)$ orqali. Shunday qilib, mavjud Reley to'liqini uchun $\Re(c/\beta), v_s, Q^{-1}, Q_s^{-1}$ qiymatlari 1–4-rasmlarda keltirilgan.

Batafsil muhokama quyidagi rasmlarda bayon etiladi.



1-rasm. Tezlikning o'zgarishi $\Re(c/\beta)$ va $v_s = \sqrt{\Re(\beta^2)}$ shuningdek sifat faktorining (Q^{-1} va Q_s^{-1}) to'yinish (σ) va chastota (ω) ga bog'liqligi. G'ovak elastik jism ($\delta_E = \delta_G = 0$) yopishqoq suyuqlik bilan to'yingan; $q_l = 10 \text{ MPa s m}^{-2}$, $K_{\text{cap}}/K_l = 0.05$



2-rasm. 1-rasm bilan bir xil, ammo bu yerda to'yinish (σ) va kapillyar bosim (K_{cap}/K_l) bilan o'zgarishi ko'rsatilgan; $\omega/(2\pi) = 100 \text{ Hz}$, $q_l = 10 \text{ MPa s m}^{-2}$.

Chastota. Reley to'liqini va kesuvchi to'liqning tezliklari ($\Re(c/\beta), v_s$) hamda so'nishlari (Q^{-1}, Q_s^{-1}) chastota ω ga bog'liqligi 1-rasmda ko'rsatilgan. Shuni ta'kidlash kerakki, Reley to'liqini ($\sigma \in (0,1)$) parametrining ixtiyoriy qiymatida mavjud emas, balki faqatgina ma'lum σ segmentda mavjud bo'ladi. Bundan tashqari, tezlik nisbati $\Re(c/\beta)$ shu segmentda nolga yaqinlashadi yoki maksimum qiymatga intiladi. Bu segment $\Re(c/\beta)$ ning chastotaga deyarli sezgir emasligini ko'rsatadi. Chastota o'zgarishi hech qanday ta'sir ko'rsatmaydi. Sifat faktorlarining katta qiymatlari Reley va kesuvchi to'liqlarning so'nishi chastotaga to'g'ridan-to'g'ri proporsional ekanligini bildiradi. Shu bilan birga, g'ovaklarning to'yinish darajasidagi juda kichik o'sish ham Reley to'liqining tezligini keskin pasaytirishi mumkin.

Kapillyar bosim. Hajmiy modul K_{cap} turli g'ovak-suyuqliklar orasidagi kapillyar bosimni bildiradi. Reley to'liqini tezligi va so'nishining $\sigma \in (0,1)$ bilan o'zgarishi 2-rasmda keltirilgan. Kapillyar bosimning ta'siri aynan shu σ -segmentda yaqqol namoyon bo'ladi. Boshqacha aytganda, kapillyar bosim kichik bo'lganda Reley to'liqini mavjud bo'lishi uchun talab qilinadigan to'yinish darajasi yuqori bo'ladi. Kutilgandek, kesuvchi to'liqin kapillyar bosim o'zgarishiga sezgir emas. Shu

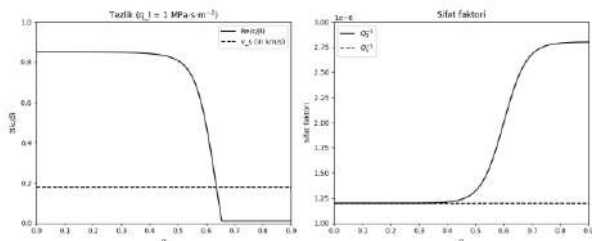
bois, K_{cap} o'zgarishi Reley to'liqining tezligi yoki so'nishiga sezilarli ta'sir ko'rsatmaydi.

G'ovak-suyuqliklarning yopishqoqligi.

Dissipatsiya koeffitsiyentlari q_g va q_l g'ovak suyuqliklaridagi yopishqoqlikni bildiradi. Fiksirlangan q_g uchun, $q_l = 1, 10, 100 \text{ MPa} \cdot \text{s} \cdot \text{m}^{-2}$ qiymatlarida Reley va kesuvchi to'liqlarning tezligi hamda sifat faktorlarining hisoblari 3-rasmda ko'rsatilgan. Grafiklardan ko'rinib turibdiki, tezliklar q_l ga deyarli javob bermaydi. Ammo Reley va kesuvchi to'liqlarning sifat faktorlarining qiymatlari q_l ga to'g'ridan-to'g'ri proporsional ravishda o'zgaradi. Demak, suyuqlik yopishqoqligidagi o'zgarish Reley to'liqining mavjud bo'lish diapazonini o'zgartirishni talab qilmaydi.

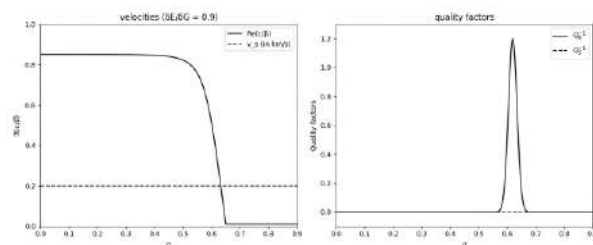
G'ovak muhitning noelastikligi.

Logarifmik dekrement qiymatlari ($\delta_G, \delta_E/\delta_G$) g'ovak muhitning noelastikligini belgilaydi. Elastik materiallarga yaqin hollarda δ_E/δ_G nisbati $[0.9, 1.5]$ oraliqda yotadi. Ma'lum bir $\delta_G = 0.11/\pi$ qiymati uchun, Reley va kesuvchi to'liqlarning tezligi va so'nishlari $\delta_E/\delta_G = 0.9$ holda hisoblangan va u 4-rasmda keltirilgan. Bu yerda g'ovak-suyuqliklar yopishqoq emas deb faraz qilingan. E'tiborli jihati shundaki, δ_E/δ_G o'zgarishi Reley to'liqining tezligi va so'nishiga ta'sir qiladi.

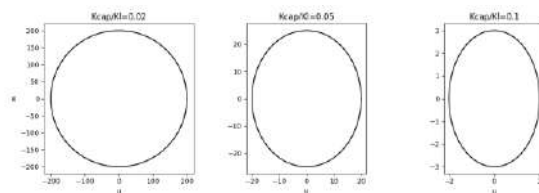


3-rasm. 1-rasm bilan bir xil, ammo bu yerda to'yinish (σ) va g'ovak-suyuqlikning dissipatsiyasi q_l bo'yicha o'zgarishi ko'rsatilgan;

$$\omega/(2\pi) = 100 \text{ Hz}, K_{cap}/K_l = 0.05$$



4-rasm. 1-rasm bilan bir xil, ammo bu yerda to'yinish (σ) va muhitning noelastikligi (δ_E/δ_G) bo'yicha o'zgarishi ko'rsatilgan. Noelastik g'ovak muhit ($\delta_G = 0.11/\pi$) yopishqoq bo'lmagan suyuqlik bilan to'yingan; $q_l = 0$, $\omega/(2\pi) = 100 \text{ Hz}$, $K_{cap}/K_l = 0.05$



5-rasm. Zarralar harakatining (u, w)

chuqurlikka ($k_R z$) va kapillyar bosimga (K_{cap}/K_l) bog'liqligi; g'ovak elastik jism yopishqoq suyuqlik bilan to'yingan;

Kapillyar bosim Reley to'liqini uchun σ -segmentga ta'sir qilmasligi mumkin. Biroq elastik muhitli g'ovak jismlardan farqli ravishda, Reley to'liq tezligi nisbati minimumga yoki nolga tushmaydi, xususan, $\delta_E/\delta_G > 1$ bo'lganda. $\Re(c/\beta)$ ning minimum qiymati σ -segmentda maksimumga yetishidan ancha oldin kuzatiladi. Ammo bu to'yinish uchun Reley to'liqining so'nishida keskin o'sish kutiladi. Umuman olganda, $\sigma > 0.6$ bo'lganda, Reley to'liq tezligi δ_E ortishi bilan oshadi.

6.2. Polarizatsiya

Ma'lumki, Reley to'liqlari zarrachalarning elliptik harakatini ifodalaydi. Shu bois, $x-z$ tekisligida zarrachalarning harakati turli chuqurliklarda hisoblanadi ($k_R z = 0, 10, 20$). Siljishlar kattaligi ($u, 0, w$) manba kuchi noma'lum bo'lganda aniqlanmaydi. Qulaylik uchun (47)–(49) tenglamalardan foydalanib, u va $w|k_A| = 1$

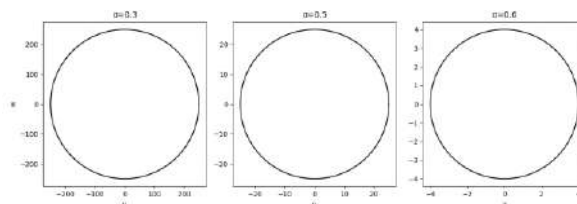
shartida hisoblangan. Shunda masshtablangan siljish vektori $(u, 0, w)/|kA|$ zarrachalarning tebranish diagrammalari (polarizatsiya diagrammalari) deb ataladi.

Aniqlik, polarizatsiya diagrammalari faqatgina nisbiy interpretatsiya uchun ishlatiladi. Hisob-kitoblardan ko‘rinadiki, istalgan chuqurlikda zarrachalarning harakati to‘lqin chastotasi (ω), g‘ovak-suyuqlik dissipatsiyasi (q_l) yoki muhitning noelastikligiga deyarli ta’sir qilmaydi. Kapillyar bosim (K_{cap}) va to‘yinish (σ) ning (u, w) polarizatsiyasiga ta’siri 5 va 6-rasmlarda keltirilgan. Batafsil izohlar quyidagicha:

Kapillyar bosim. Masshtablangan siljish vektori $(u, 0, w)/|kA|$ $\sigma = 0.5$ uchun hisoblangan va konturlari 5-rasmda ko‘rsatilgan. 2-rasmdan ko‘rinib turibdiki, bu to‘yinish qiymatida Reley to‘lqini mavjud bo‘lishi ta’minlangan. Kutilganidek, zarrachalarning harakati chuqurlik bilan so‘nadi va so‘nish darajasi chuqurlashgan sari ortadi. Zarracha harakatining elliptikligi ham chuqurlik bilan o‘zgaradi. Xulosa qilib aytganda, sirt yoki muhit ichida bo‘lishidan qat’iy nazar, kapillyar bosim ortishi bilan zarracha harakati kamayadi. Bundan tashqari, katta chuqurliklar bundan mustasno, kapillyar bosim ortishi zarracha harakati tezligining pasayishiga olib keladi.

To‘yinish. G‘ovak elastik jism yopishqoq suyuqlik bilan qisman to‘yingan holda qaraladi. To‘yinish parametri $\sigma = 0.3, 0.5, 0.6$ qiymatlarida olingan. Reley to‘lqinining mavjudligi ushbu qiymatlar uchun ta’minlangan (1–3-rasmlarda ko‘rsatilgan). Masshtablangan siljishlar hisoblanib, konturlari 6-rasmda berilgan. Shuni kuzatish mumkinki, σ ortishi bilan zarrachalarning harakati pasayadi. So‘nish darajasi chuqurlik ortgan sari ortib boradi. Bundan tashqari, ushbu to‘yinish uchun polarizatsiyada ozgina o‘zgarish kuzatilishi mumkin. ga bog‘liq so‘nish darajasi

chuqurlik bilan kuchayadi. Biroq katta chuqurliklarda elliptiklikdagi o‘zgarish kuchli so‘nish sharoitida muhim ahamiyat kasb etmaydi.



6-rasm. 5-rasm bilan bir xil, ammo bu yerda chuqurlik ($k_R z$) va suyuqlikning to‘yinish darajasi bo‘yicha o‘zgarishi ko‘rsatilgan;

7. Xulosa

Mazkur ishda ikki aralashmaydigan suyuqlik aralashmasi bilan to‘yingan g‘ovak jismlarda Reley to‘lqinining eksponensial o‘zgarishi tadqiq qilindi. Agar g‘ovak suyuqliklardan biri havo yoki siyrak gaz deb qaralsa, model qisman to‘yingan g‘ovak muhitni ifodalaydi. Ko‘rib chiqilgan muhit yo‘qotishsiz emas: energiya dissipatsiyasi qattiq jismda energiya dissipatsiyasi yo‘qotishlar hamda g‘ovaklardagi yopishqoqlik tufayli yuzaga keladi. Dissipatsiya mavjud hollarda irratsional sekulyar tenglama Reley to‘lqinining mavjudligi va tarqalishini belgilaydi. Ushbu tenglama polinom shakliga keltirilib, sonli usullar orqali ildizlari topildi. Ildizlar chuqurlik bo‘yicha siljishlarning so‘nishini ta’minlash sharti asosida tanlab olindi. Shu tarzda olingan ildizlar qisman to‘yingan dissipativ g‘ovak jismlarda Reley to‘lqinining murakkab tezligini aniqlaydi.

Ushbu usulni namoyish etish uchun maxsus sonli misol ko‘rib chiqildi. Dissipativ g‘ovak jismlar uchun o‘tkazilgan hisoblash va tahlillar asosida quyidagi foydali xulosalar chiqarildi:

- Reley to‘lqini har qanday qiymatida emas, faqat muayyan segmentda mavjud. Qattiq g‘ovak muhitda tezlik segmentning yuqori chegarasida nolga yaqinlashib, kichik ortish ham keskin kamayishga olib keladi.

- G'ovak elastik muhitda tezlik nolga tushmaydi, lekin maksimumga yetishidan oldin minimumga erishadi va bu keskin so'nish bilan birga sodir bo'ladi.

- To'yinish ortishi bilan zarracha harakati chuqurlikda tezroq so'nadi; elliptik trayektoriya yuzada ham, ichkarida ham saqlanib, ortishi bilan so'nish kuchayadi.

- Reley to'lqinining sifat faktori chastota va suyuq faza yopishqoqligiga proporsional, ammo zarracha harakati ularga deyarli bog'liq emas.

- Kapillyar bosim kichik bo'lganda, Reley to'lqini mavjud bo'lishi uchun yuqori to'yinish talab qilinadi. Bosim ortganda esa elliptik trayektoriya qisqarib, zarrachalar harakati chuqurlashgan sari qiyshayadi.

G'ovaklar ikki fazali aralashma bilan to'yinganda, muhitdagi real vaqt o'zgarishlarini tadqiq etish uchun umumlashtirilgan va yanada real model olinadi. Taklif etilgan algoritmnining moslashuvchanligi yopishqoq suyuqliklar bilan to'yingan g'ovak muhit va noelastik karkas modellarida ham tasdiqlandi.

G'ovaklardagi suyuqlik ulushining o'zgaruvchanligi muhitni deyarli quruqdan to'liq to'yingangacha ifodalash imkonini beradi. Seysmik qo'llanmalarda manba va qabul qiluvchilar ko'pincha sirtga yaqin joylashgani bois, to'yinish darajasiga bog'liq Reley to'lqinlarini tahlil qilish qidiruv seysmologiyasi hamda zilzila oldi jarayonlarini tushunishda muhim ahamiyat kasb etadi.

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TRANSLATION-INVARIANT GIBBS MEASURES FOR THE BLUME–CAPEL MODEL ON THE CAYLEY TREE OF ORDER THREE

N.M.Khatamov¹, M.A.Kodirova²

¹Namangan state university, Uzbekistan, Namangan city, Davlatobod district, Yangi Shahar street, house 30

²Andijan state university, Uzbekistan, 170100, Andijan city, Universitet street, house 129

Submitted September 2, 2025; accepted for publication October 26, 2025

Annotatsiya. Ushbu maqolada k –tartibli Keli daraxtida Blum-Kapel modeli uchun funksional tenglama keltirib chiqilgan. Shuningdek, daraxt tartibi $k = 3$ bo'lgan holatda va qurilgan funksional tenglamada $z_1 = z_2$ sharti ostida translatsion-invariant Gibbs o'lchovlari tuzilgan. Tadqiqot natijalariga ko'ra, kritik qiymat $\theta_{cr} \approx 0.20788$ aniqlangan va $0 < \theta < \theta_{cr}$ oraliqda uchta, $\theta = \theta_{cr}$ da ikkita, va $\theta > \theta_{cr}$ da faqat bitta translatsion-invariant Gibbs o'lchovi mavjudligi ko'rsatilgan.

Kalit so'zlar: Keli daraxti, konfiguratsiya, Blum-Kapel modeli, Gibbs o'lchovi, translyatsion-invariant o'lchovlar.

Аннотая. В данной статье выведено функциональное уравнение для модели Блюма-Капеля на дереве Кэли порядка k . Более того, в случае порядка дерева $k = 3$ и при условии $z_1 = z_2$ в построенном функциональном уравнении установлены трансляционно-инвариантные меры Гиббса. Результаты исследования позволяют определить критическое значение $\theta_{cr} \approx 0.20788$ и показывают, что существует три трансляционно-инвариантные меры Гиббса при $0 < \theta < \theta_{cr}$, две при $\theta = \theta_{cr}$ и только одна при $\theta > \theta_{cr}$.

Ключевые слова: дерево Кэли, конфигурация, модель НС- Блюма-Капеля, мера Гиббса, трансляционно-инвариантные меры.

Abstract. In this paper, a functional equation for the Blum-Capel model on a k -order Cayley tree is derived. Furthermore, for the case of tree order $k = 3$ and under the condition $z_1 = z_2$ in the constructed functional equation, the translation-invariant Gibbs measures are established. The results of the study identify a critical value $\theta_{cr} \approx 0.20788$ and demonstrate that there exist three translation-invariant Gibbs measures for $0 < \theta < \theta_{cr}$, two for $\theta = \theta_{cr}$, and only one for $\theta > \theta_{cr}$.

Keywords: Cayley tree, configuration, Blume-Capel model, Gibbs measure, translation-invariant measures

1. Introduction.

The Gibbs measure is a fundamental law determining the probability of a microscopic state of a given physical system and it plays an important role in determining the existence of a phase transition of a physical system, since each Gibbs measure is associated with one phase of the physical system, and if a Gibbs measure is nonunique, then it is said that there is a phase transition. It is well known that the set of all limit Gibbs measures forms a nonempty convex compact subset of the set of all probability measures and each point (i.e., Gibbs measure) of this convex set can be uniquely expanded to its extreme points. Therefore, it is interesting to describe all extreme points of this convex set, i.e., the extreme Gibbs measures (see [2,7,10]). Many papers are devoted to the study of limit Gibbs measures on a Cayley tree for such models of statistical physics as Ising model, Potts model, HC model and SOS model (see, for example, [1,3-6,9]). In particular, in [6], the set of translation-invariant Gibbs measures for the ferromagnetic q –state Potts model was fully described and it was

proved that the number of translation-invariant measures can be up to $2^q - 1$, and in [5], the extremality problem was studied for these measures. In [9], Gibbs measures for three state HC models were studied on a Cayley tree order $k \geq 1$ and the nonuniqueness of the translation-invariant Gibbs measure was proved. Moreover, the areas where the measures are (not) extreme are given. In the monograph [8], the results on limit Gibbs measures can be found in more detail.

In [11], the translation-invariant Gibbs measures for the Blume–Capel model with a wand on a Cayley tree of order k are studied. The exact critical value $\theta_{cr} = 1$ is found, such that there exists a unique translation-invariant Gibbs measure for $\theta \geq \theta_{cr}$, and exactly three translation-invariant Gibbs measures for $0 < \theta < \theta_{cr}$. In addition, the problem of extremality and non-extremality of these measures is investigated.

This paper is organized as follows. In Section 2, we present the basic definitions and known facts. In Section 3, we prove a theorem that ensures the condition of consistency of a measure. In Section 4,

we construct translation-invariant Gibbs measures for $k = 3$ and $z_1 = z_2$.

2. Preliminary information

A Cayley tree $\Gamma^k = (V, L)$ of order $k \geq 1$ is an infinite tree, i.e., a graph without cycles such that each vertex has precisely $k + 1$ edges, where V is the set of vertices of the graph Γ^k , L is the set of its edges. Let i be an incidence function associating each edge $l \in L$ to its endpoints $x, y \in V$. If $i(l) = \{x, y\}$, then x and y are called the nearest neighbors of a vertex and can be written as $\langle x, y \rangle$. The distance $d(x, y)$, $x, y \in V$ on the Cayley tree is defined as

$$d(x, y) = \min\{d|\exists x = x_0, x_1, \dots, x_{d-1}, x_d = y; \langle x_0, x_1 \rangle, \dots, \langle x_{d-1}, x_d \rangle\}.$$

It is known that a Cayley tree can be represented in the form of a group G_k obtained as the free product of $k + 1$ cyclic groups of the second order with generators $a_1, a_2, \dots, a_{k+1}$ [8].

We consider a model in which spin variables take values from the set $\Phi = \{-1, 0, 1\}$. We then define a configuration σ on V as function $x \in V \rightarrow \sigma(x) \in \Phi$. The set of all configurations coincides with $\Omega = \Phi^V$. Let $A \subset V$. We denote the space of configurations defined on a set A by Ω_A . The Hamiltonian of the Blume-Capel model is given by the formula

$$H(\sigma) = J \sum_{\langle x, y \rangle} (\sigma(x) - \sigma(y))^2, \quad (1)$$

where $J > 0$.

For a fixed $x^0 \in V$, we write $x < y$ if a path from x^0 to y runs through x .

Denote

$$W_n = \{x \in V : d(x^0, x) = n\}, \quad V_n = \{x \in V : d(x^0, x) \leq n\}.$$

A point y is called a “direct derivate” of the point x if $x < y$ and $d(x, y) = 1$.

For $x \in G_k$, by $S(x)$ we denote a set of “direct derivatives” of the point $x \in V$.

Let $h : x \mapsto h_x = (h_{-1,x}, h_{0,x}, h_{1,x})$ be a vector-valued function on $x \in V \setminus \{x^0\}$. We consider the probability measure $\mu^{(n)}$ on Ω_A , $\mu^n(\sigma_n) = Z_n^{-1} \exp\{-\beta H(\sigma_n) + \sum_{x \in W_n} h_{\sigma(x),x}\}$. (2) Here $\sigma \in \Omega_{V_n}$, and Z_n is a normalization factor,

$$Z_n = \sum_{\bar{\sigma}_n \in \Omega_{V_n}} \exp\left\{-\beta H(\bar{\sigma}_n) + \sum_{x \in W_n} h_{\bar{\sigma}(x),x}\right\},$$

where $h_{\bar{\sigma},x} \in R$.

The probability measure $\mu^{(n)}$ is said to be consistent if for all $n \geq 1$ and any $\sigma_{n-1} \in \Omega_{V_{n-1}}$:

$$\sum_{\sigma^{(n)}} \mu^{(n)}(\sigma_{n-1}, \sigma^{(n)}) = \mu^{(n-1)}(\sigma_{n-1}) \quad (3)$$

In this case, there is a unique measure μ on Ω_V such that

$$\mu(\{\sigma | V_n = \sigma_n\}) = \mu^{(n)}(\sigma_n)$$

for all $n \geq 1$ and any $\sigma_n \in \Omega_{V_n}$.

3. System of Functional Equations

The following theorem contains necessary and sufficient conditions for $h_{i,x}$ under which relation (3) is true:

Theorem 1. Suppose that $k \geq 2$. The probability distribution $\mu^{(n)}(\sigma_n)$, $n = 1, 2, \dots$, in (2) is consistent if and only if, for any $x \in V$, the following equations are true:

$$\begin{cases} z_{1,x} = \prod_{y \in S(x)} \frac{\theta^4 z_{-1,y} + \theta + z_{1,y}}{\theta z_{-1,y} + 1 + \theta z_{1,y}}, \\ z_{-1,x} = \prod_{y \in S(x)} \frac{z_{-1,y} + \theta + \theta^4 z_{1,y}}{\theta z_{-1,y} + 1 + \theta z_{1,y}}, \end{cases} \quad (4)$$

where $\theta = \exp\{-J\beta\}$, $\beta = \frac{1}{T}$, $z_{i,x} = \exp\{h_{i,x} - h_{0,x}\}$, and $i = -1, 1$.

Proof. Necessary. By virtue of the consistency condition (3), we get

$$\begin{aligned} \frac{Z_{n-1}}{Z_n} \sum_{\omega_n \in \Omega_{W_n}} \prod_{x \in W_{n-1}} \prod_{y \in S(x)} \exp\{-J\beta(\sigma_{n-1}(x) - \omega_n(y))^2 + h_{\omega_n(y),y}\} \\ = \prod_{x \in W_{n-1}} \exp(h_{\sigma_n(x),x}), \end{aligned} \quad (5)$$

Where $\sigma(x) \in \Phi$.

In the case where $\sigma_{n-1}(x) = -1$, $\sigma_{n-1}(x) = 0$, and $\sigma_{n-1}(x) = 1$, we rewrite relation (5) in the form

$$\begin{aligned} \exp(h_{+1,x} - h_{0,x}) \\ = \prod_{y \in S(x)} \frac{\sum_{\omega_n \in \Omega_{W_n}} \exp\{-J\beta(1 - \omega_n(y))^2 + h_{\omega_n(y),y}\}}{\sum_{\omega_n \in \Omega_{W_n}} \exp\{-J\beta\omega_n^2(y) + h_{\omega_n(y),y}\}} \\ \exp(h_{-1,x} - h_{0,x}) \\ = \prod_{y \in S(x)} \frac{\sum_{\omega_n \in \Omega_{W_n}} \exp\{-J\beta(-1 - \omega_n(y))^2 + h_{\omega_n(y),y}\}}{\sum_{\omega_n \in \Omega_{W_n}} \exp\{-J\beta\omega_n^2(y) + h_{\omega_n(y),y}\}}. \end{aligned}$$

Therefore,

$$\begin{aligned} \exp(h_{1,x} - h_{0,x}) \\ = \prod_{y \in S(x)} \frac{\exp\{-4J\beta\} \exp\{h_{-1,y} - h_{0,y}\} + \exp\{-J\beta\} + \exp\{h_{1,y} - h_{0,y}\}}{\exp\{-J\beta\} \exp\{h_{-1,y} - h_{0,y}\} + 1 + \exp\{-J\beta\} \exp\{h_{1,y} - h_{0,y}\}} \\ \exp(h_{-1,x} - h_{0,x}) \\ = \prod_{y \in S(x)} \frac{\exp\{-4J\beta\} \exp\{h_{1,y} - h_{0,y}\} + \exp\{-J\beta\} + \exp\{h_{-1,y} - h_{0,y}\}}{\exp\{-J\beta\} \exp\{h_{-1,y} - h_{0,y}\} + 1 + \exp\{-J\beta\} \exp\{h_{1,y} - h_{0,y}\}}. \end{aligned}$$

This yields (4).

Sufficiency. Assume that (4) holds. Then, for some function $b(x) > 0$, $x \in V$, the following relation is true:

$$\prod_{y \in S(x)} \sum_{u \in \{-1, 0, 1\}} a_{t,u} \exp\{-J\beta(t - u)^2 + h_{u,y}\} = b(x) \exp(h_{t,x}), \quad t = -1, 0, 1. \quad (6)$$

We have

$$(3) = \frac{1}{Z_n} \exp(-\beta H(\sigma_{n-1})) \times \prod_{x \in W_{n-1}} \prod_{y \in S(x)} \sum_{u \in \{-1, 0, 1\}} \exp\{-J\beta(\sigma_{n-1}(x) - u)^2 + h_{u,y}\}. \quad (7)$$

By using (6) and introducing the notation

$$B_n(x) = \prod_{x \in W_{n-1}} b(x),$$

we get the following relation from (7):

$$(6) = \frac{B_{n-1}}{Z_n} \times \exp(-\beta H(\sigma_{n-1})) \prod_{x \in W_{n-1}} \exp(h_{\sigma_{n-1}(x), x}). \quad (8)$$

Since $\mu^{(n)}$, $n \geq 1$, is the probability, we obtain

$$\sum_{\sigma_{n-1} \in \Omega_{V_{n-1}}} \sum_{\omega_n \in \Omega_{W_n}} \mu^{(n)}(\sigma_{n-1}, \omega_n) = 1.$$

Therefore, by using (8), we arrive at the equality

$$Z_{n-1} B_{n-1} = Z_n.$$

Hence, relation (3) is true.

Theorem 1 is proved.

4. Translation-Invariant Gibbs Measures

The translation-invariant Gibbs measures correspond to the solutions of (4) with $z_{i,x} = z_i$ for all $x \in V$ and $i = -1, 1$. For convenience, we denote $z_1 = z_1$ and $z_{-1} = z_2$.

Then the system of equations (4) takes the following form

$$\begin{cases} z_1 = \left(\frac{\theta^4 z_2 + \theta + z_1}{\theta z_1 + \theta z_2 + 1} \right)^k, \\ z_2 = \left(\frac{\theta^4 z_1 + \theta + z_2}{\theta z_1 + \theta z_2 + 1} \right)^k. \end{cases} \quad (9)$$

Let $z_1 = z_2 = z$ and $k = 3$. Thus, it follows from (9) that

$$z = \left(\frac{(1+\theta^4)z + \theta}{2\theta z + 1} \right)^3. \quad (10)$$

We now consider the solution of equation (10) in the case $k = 3$. By introducing the substitution $\sqrt[3]{z} = t$, we obtain that equation (10) takes the following form:

$$t^4 + at^3 + bt^2 + ct + d = 0, \quad (11)$$

where $a = -\frac{1+\theta^4}{2\theta}$, $b = 0$, $c = \frac{1}{2\theta}$, $d = -\frac{1}{2}$.

The discriminant of the resulting quartic equation is given by:

$$\Delta(\theta) = (-27\theta^{18} - 108\theta^{14} + 4\theta^{12} - 150\theta^{10} + 780\theta^8 - 2132\theta^6 + 780\theta^4 - 123\theta^2 + 4)/(64\theta^6).$$

By solving the equation $\Delta(\theta) = 0$, we find $\theta_{cr} \approx 0.206877$. The following theorem is known from algebra.

Theorem 2. For the quartic equation, $t^4 + at^3 + bt^2 + ct + d = 0$, the following statements hold depending on the sign of its discriminant Δ :

- If $\Delta > 0$, then the quartic equation has either four distinct real roots or four distinct complex roots;
- If $\Delta < 0$, then the quartic equation has two distinct real roots and two distinct complex roots;
- If $\Delta = 0$, then the quartic equation has at least two mutually equal roots.

According to Ferrari's method, solving the equation $t^4 + at^3 + bt^2 + ct + d = 0$ consists in expressing its left-hand side as a difference of two squares. As a result, the problem of solving the given equation is reduced to solving two quadratic equations. For this purpose, we can rewrite the left-hand side of the equation as follows:

$$\left(t^2 + \frac{a}{2}t + \frac{y}{2}\right)^2 - \left[\left(\frac{a^2}{4} + y - b\right)t^2 + \left(\frac{ay}{2} - c\right)t + \left(\frac{y^2}{4} - d\right)\right] = 0. \quad (12)$$

Here y is an auxiliary unknown chosen so that the expression inside the square brackets becomes the square of a linear binomial. It is well known that for a quadratic $At^2 + Bt + C = 0$ to be the square of a linear binomial it is necessary and sufficient that its discriminant vanish, i.e., $B^2 - 4AC = 0$.

Accordingly, the coefficients are given by

$$A = y + \frac{(1+\theta^4)^2}{16\theta^2}, B = -\frac{1+\theta^4}{4\theta}y - \frac{1}{2\theta}, C = \frac{y^2}{4} + \frac{1}{2},$$

$$B^2 - 4AC = \left(-\frac{1+\theta^4}{4\theta}y - \frac{1}{2\theta}\right)^2 - 4\left(y + \frac{(1+\theta^4)^2}{16\theta^2}\right)\left(\frac{y^2}{4} + \frac{1}{2}\right) = 0$$

which represents a cubic equation in y .

It is well known that a cubic equation always has at least one real root. Using Cardano's method, the real solution of the equation can be expressed as follows:

$$y_1 = \frac{\sqrt[3]{\alpha + 12\sqrt{3\beta}}}{12\theta} + \frac{\theta^4 - 8\theta^2 + 1}{\theta^3 \sqrt[3]{\alpha + 12\sqrt{3\beta}}}$$

where $\alpha = -108\theta^9 - 216\theta^5 + 108\theta$, $\beta = 27\theta^{18} + 108\theta^{14} - 4\theta^{12} + 150\theta^{10} - 780\theta^8 + 2132\theta^6 - 780\theta^4 + 123\theta^2 - 4$.

If the number of sign changes among the coefficients of a polynomial is k , then the number of its positive real roots is either $k, k-2, k-4, \dots$. Hence, for equation (11) with $k = 3$, the number of real roots is either three or one. For the quadratic expression in equation (12), the following cases are possible:

Case 1. If $B = 0$, then equation (11) has one positive real root;

Case 2. If $B < 0$, the quadratic expression becomes a perfect square of the form $(\sqrt{A}t - \sqrt{C})^2$;

Case 3. If $B > 0$, the quadratic expression becomes a perfect square of the form $(\sqrt{A}t + \sqrt{C})^2$.

Let us examine each case in detail.

Case 1. For $\theta^* \approx 1,2889$, we have $B = 0$. In this case, equation (11) has a single solution, which is approximately $t \approx 1,4294$.

Case 2. Let $B < 0$, then $0 < \theta < \theta^*$.

For $0 < \theta < \theta^*$, equation (8) can be rewritten as:

$$\left(t^2 + \frac{a}{2}t + \frac{y_1}{2}\right)^2 - (\sqrt{A}t - \sqrt{C})^2 = 0,$$

which factors into

$$\left(t^2 + \left(\frac{a}{2} - \sqrt{A}\right)t + \frac{y_1}{2} + \sqrt{C}\right)\left(t^2 + \left(\frac{a}{2} + \sqrt{A}\right)t + \frac{y_1}{2} - \sqrt{C}\right) = 0.$$

As a result, we obtain two quadratic equations:

$$t^2 + \left(\frac{a}{2} + \sqrt{A}\right)t + \frac{y_1}{2} - \sqrt{C} = 0, \quad (13)$$

$$t^2 + \left(\frac{a}{2} - \sqrt{A}\right)t + \frac{y_1}{2} + \sqrt{C} = 0. \quad (14)$$

We now examine the roots of equations (13)–(14) for $0 < \theta < \theta^*$.

(i) Let $\theta \in (0, \theta_{cr})$. For the roots t_0 and t_1 of equation (13), in this interval we have $t_0 t_1 = \frac{y_1}{2} - \sqrt{C} = \frac{y_1}{2} - \sqrt{\frac{y_1^2}{4} + \frac{1}{2}} < 0$. Thus, the roots of equation (13) have opposite signs, and its positive root is

$$t_1 = \frac{-\frac{a}{2} - \sqrt{A} + \sqrt{\left(\frac{a}{2} + \sqrt{A}\right)^2 - 4\left(\frac{y_1}{2} - \sqrt{C}\right)}}{2}.$$

Accordingly, the solution of equation (10) is $z_1 = t_1^3$.

Let us verify that the following conditions hold for equation (14):

$$\begin{cases} t_2 + t_3 > 0, \\ t_2 t_3 > 0, \\ D = \left(\frac{a}{2} - \sqrt{A}\right)^2 - 4\left(\frac{y_1}{2} + \sqrt{C}\right) > 0. \end{cases}$$

Here, $t_2 t_3 = \frac{y_1}{2} + \sqrt{C} = \frac{y_1}{2} + \sqrt{\frac{y_1^2}{4} + \frac{1}{2}} > 0$ which is always satisfied. Therefore, the roots of equation (14) have the same sign.

Let us determine the sign of the expression $t_2 + t_3 = -\frac{a}{2} + \sqrt{A}$. In the interval $(0, \theta_{cr})$, since $y_1 > 0$ (see Fig. 1), we have $\sqrt{A} = \sqrt{\frac{a^2}{4} + y_1} > \frac{a}{2}$, which is always satisfied. It follows that $t_2 + t_3 > 0$.

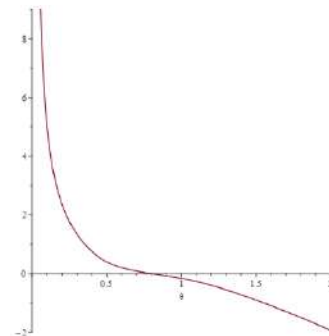


Fig 1. Graph of y_1 in the interval $(0, \theta_{cr})$.

The graph of D in the interval $(0, \theta_{cr})$ allows an easy verification that $D > 0$ (see Fig. 2).

Furthermore,

$$\lim_{\theta \rightarrow 0^+} D(\theta) = +\infty \text{ and } \lim_{\theta \rightarrow \theta_{cr}^-} D(\theta) = 0.$$

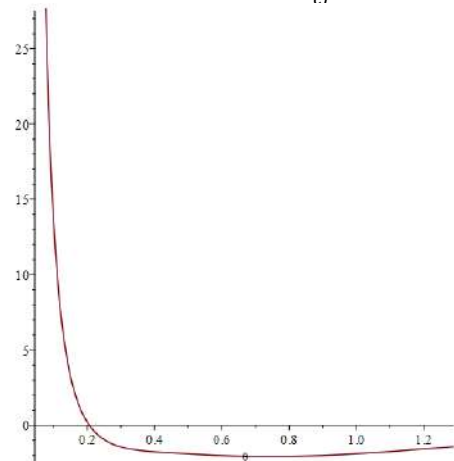


Fig 2. Graph of D in the interval $(0, \theta^*)$.

Thus, in the interval $(0, \theta_{cr})$, the roots of equation (14) are positive and are given by:

$$t_2 = \frac{-\frac{a}{2} + \sqrt{A} + \sqrt{\left(\frac{a}{2} - \sqrt{A}\right)^2 - 4\left(\frac{y_1}{2} + \sqrt{C}\right)}}{2},$$

$$t_3 = \frac{-\frac{a}{2} + \sqrt{A} - \sqrt{\left(\frac{a}{2} - \sqrt{A}\right)^2 - 4\left(\frac{y_1}{2} + \sqrt{C}\right)}}{2}.$$

Accordingly, the solutions of equation (10) are $z_2 = t_2^3$, $z_3 = t_3^3$.

It follows that, $\theta \in (0, \theta_{cr})$, equation (10) has three positive roots.

(ii) Let us consider $\theta = \theta_{cr}$. In this case, equation (11) has two positive solutions, with approximate values $t_1 \approx 0,216$ and $t_2 \approx 1.571$.

Accordingly, the solutions of equation (10) are $z_1 = t_1^3$ and $z_2 = t_2^3 = t_3^3$.

(iii) For $\theta \in (\theta_{cr}, \theta^*)$, equation (13) retains a positive root t_1 , yielding the solution of equation (10) as $z_1 = t_1^3$. In contrast, equation (14) has a negative discriminant $D < 0$ (see Fig. 2), implying the absence of real solutions. Moreover, $\lim_{\theta \rightarrow \theta_{cr}^+} D(\theta) = 0$ and $\lim_{\theta \rightarrow \theta^*-} D(\theta) < 0$.

Case 3. Let $B > 0$, which corresponds to the interval $\theta \in (\theta^*, +\infty)$. In this range, equation (12) can be rewritten as

$$\left(t^2 + \frac{a}{2}t + \frac{y_1}{2}\right)^2 \mp (\sqrt{A}t + \sqrt{C})^2 = 0,$$

which factors into two quadratic equations:

$$\left(t^2 + \left(\frac{a}{2} + \sqrt{A}\right)t + \frac{y_1}{2} + \sqrt{C}\right)\left(t^2 + \left(\frac{a}{2} - \sqrt{A}\right)t + \frac{y_1}{2} - \sqrt{C}\right) = 0.$$

Thus, we obtain the following two quadratic equations:

$$t^2 + \left(\frac{a}{2} - \sqrt{A}\right)t + \frac{y_1}{2} - \sqrt{C} = 0, \quad (15)$$

$$t^2 + \left(\frac{a}{2} + \sqrt{A}\right)t + \frac{y_1}{2} + \sqrt{C} = 0. \quad (16)$$

For equation (15) in the interval $\theta \in (\theta^*, +\infty)$, the roots t_0 and t_1 satisfy $t_0 t_1 < 0$, i.e., they have opposite signs. The positive root is

$$t_* = \frac{-\frac{a}{2} + \sqrt{A} + \sqrt{\left(\frac{a}{2} - \sqrt{A}\right)^2 - 4\left(\frac{y_1}{2} - \sqrt{C}\right)}}{2}$$

yielding the solution of equation (10) as $z_1 = t_*^3$.

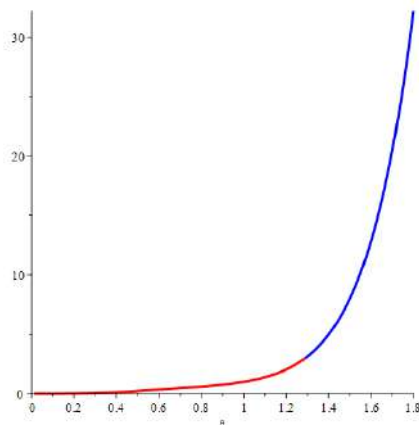


Fig 3. The graph of the solution z_1 .

Since equation (15) already provides two real roots, there is no need to consider equation (16) (Theorem 2). Indeed, for $\theta \in (\theta^*, +\infty)$, the discriminant of equation (11) satisfies $\Delta < 0$, so only two roots are real, which are already captured by (15).

Therefore, the solution z_1 of equation (10) can be expressed as a piecewise continuous function (since $B = 0$ implies $A = 0$ and $t_1 = t_*$) (see Fig. 3):

$$z_1 = \begin{cases} t_1^3, & \text{if } \theta \in (0, \theta^*), \\ t_*^3, & \text{if } \theta \in (\theta^*, +\infty). \end{cases}$$

This completes the proof of the theorem.

Theorem 3. Let $k = 3$ and $z_1 = z_2 = z$. Then for the Blume-Capel model in the form of (1), there exists a critical value $\theta_{cr} \approx 0,206877$, such that:

- 1) for $0 < \theta < \theta_{cr}$ there are three translation-invariant Gibbs measures: μ_1, μ_2 and μ_3 ;
- 2) for $\theta = \theta_{cr}$ there are two translation-invariant Gibbs measures: μ_1, μ_2 ;
- 3) for $\theta > \theta_{cr}$ there exists only one translation-invariant Gibbs measure: μ_1 .

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Mualliflar haqida ma'lumot

1. **Xatamov Nosirjon Muydinovich**– f.-m.f.d., Namangan davlat universiteti Matematika kafedrasi mudiri. Tel.: +99890-555-57-34; e-mail: nxatamov@mail.ru
2. **Qodirova Malika Abdupatto qizi** – Andijon davlat universiteti, Algebra va analiz kafedrasi tayanch doktoranti. Tel.: +99899-295-12-24; e-mail: malika.kodirova24@gmail.com

GIPERBOLIK TIPDAGI TENGLAMA UCHUN BOSHLANG'ICH SHARTDA YUQORI TARTIBLI HOSILA BERILGAN ARALASH MASALA

M.S. Azizov¹, M.Z. Obidjonova²

Farg'ona davlat universiteti

Andijon davlat universiteti

muzaffar.azizov.1988@mail.ru

2025-yil 3-sentyabrda berilgan; 2025-yil 19-oktyabrda chop etish uchun qabul qilingan

Annotasiya. Mazkur ishda to'g'ri to'rtburchakli sohada giperbolik tipdagi tenglama uchun aralash masala ko'rib chiqilgan. Bu yerda boshlang'ich shartlardan biri sifatida noma'lum funksiyaning yuqori tartibli vaqt bo'yicha hosilasi berilgan bo'lib, bu holat an'anaviy masalalardan farq qiladi. Masalani yechish uchun Furiye usuli qo'llanilgan. Ushbu yondashuv yordamida yechim trigonometrik qator ko'rinishida ifodalanadi va Furiye koeffitsientiga mos oddiy differensial tenglama yechilib, masalaning yechimi yagona va mavjudligi isbotlanadi.

Kalit so'zlar: giperbolik tip, boshlang'ich shart, chegaraviy shart, Furiye usuli, xos son va xos funksiyalar, Bessel funksiyasi, Furiye qatori.

Аннотация. В данной работе рассматривается смешанная задача для гиперболического типа уравнения в прямоугольной области. Здесь одна из начальных условий задана через производную неизвестной функции по времени высшего порядка, что отличает данную постановку от классических задач. Для решения задачи применен метод Фурье. С помощью данного подхода решение выражается в виде тригонометрического ряда, решается обычное дифференциальное уравнение для коэффициента Фурье, и доказана единственность и существование решения.

Ключевые слова: гиперболический тип, начальное условие, граничное условие, метод Фурье, собственное число и собственные функции, функция Бесселя, ряд Фурье.

Abstract. In this work, a mixed problem for a hyperbolic-type equation in a rectangular domain is considered. One of the initial conditions is given in terms of a higher-order time derivative of the unknown function, which distinguishes this formulation from classical problems. The Fourier method is applied to solve the problem. Using this approach, the solution is expressed as a trigonometric series, the ordinary differential equation for the Fourier coefficient is solved, and the uniqueness and existence of the solution are established.

Keywords: hyperbolic type, initial condition, boundary condition, Fourier method, eigenvalue and eigenfunctions, Bessel function, Fourier series.

Kirish

Matematik fizikaning asosiy bo'limlaridan biri bo'lib, giperbolik tipdagi differensial tenglamalarni o'rganish alohida o'rin tutadi. Bunday tenglamalar to'liq tarqalishi, tebranishlar, akustika, elektromagnit to'liqlar, mexanik sistemalardagi impulslar kabi fizik jarayonlarni modellashtiradi. A. N. Tihonov va A. A. Samarskiy asarlarida bu tenglamalar matematik fizikaning klassik tenglamalari sifatida qaralib, ularning yechimlari, mavjudlik va turg'unlik shartlari chuqur o'rganilgan [1]. Shuningdek, R. Courant va D. Gilbert [2] asarlarida ham giperbolik tenglamalarning umumiy xossalari, boshlang'ich va chegaraviy shartlarning fizik mazmuni keng tahlil etilgan.

Giperbolik tenglamalar uchun boshlang'ich-chegaraviy masalalarda

ko'pincha boshlang'ich shart sifatida funksiya va uning birinchi tartibli vaqt bo'yicha hosilasi beriladi. Ammo ayrim amaliy holatlarda, masalan elastiklik nazariyasi, seysmik to'liqlar tarqalishi yoki elektromexanik tizimlarda, noma'lum funksiyaning yuqori tartibli vaqt bo'yicha hosilasi ham berilishi zarur bo'ladi. Bunday shartli masalalar klassik Koshi masalalariga nisbatan murakkabroq bo'lib, ularning yechimlari qo'shimcha silliqlik, moslik va integrallanish shartlarini talab etadi [3, 4]. Shuning uchun bu turdagi masalalar matematik jihatdan ham, fizik jihatdan ham dolzarb hisoblanadi.

E'tiborga molikki, $\gamma = 0$ holatida qaralayotgan giperbolik tenglama klassik tor tebranish tenglamasining xususiy holatiga aylanadi. Shu bois, mazkur umumlashirilgan

masala nafaqat nazariy, balki amaliy ahamiyatga ham ega bo'lib, u tor, sim yoki chiziqli elastik elementlarning tebranishlarini tasvirllovchi model sifatida ham ishlatiladi [1, 2]. Boshqacha aytganda, bu ish tor tebranish tenglamasini o'z ichiga olgan kengroq sinfdagi aralash (boshlang'ich-chegaraviy) masalani o'rganishga bag'ishlangan.

Mazkur yo'nalishdagi masalalar bo'yicha xorijiy adabiyotlarda G. F. D. Duff [5] va T. Shirota hamda K. Asano [6] ishlarida giperbolik tipdagi tenglamalar uchun aralash masalalar tahlil qilingan. Ularning tadqiqotlarida yuqori tartibli hosilalar ishtirokidagi boshlang'ich shartlar uchun yechimlarning mavjudligi va yagona yechim shartlari o'rganilgan.

Shuningdek, o'zbek olimlari orasida D. Amanov tomonidan ham shunga yaqin yo'nalishdagi ishlar olib borilgan. Uning ba'zi ishlari boshlang'ich shartlarda noma'lum funksiyaning yuqori tartibli hosilasi ishtirok etishi bilan masalalarni o'rganishga bag'ishlangan bo'lib, bu tadqiqot biz ko'rib chiqayotgan masalaga yaqin yo'nalishda joylashgan [7].

Shu tariqa, mazkur ishda giperbolik tipdagi tenglama uchun boshlang'ich-chegaraviy masala o'rganiladi. Masalaning o'ziga xos jihati shundaki, unda boshlang'ich shartlardan biri sifatida noma'lum funksiyaning yuqori tartibli vaqt bo'yicha hosilasi berilgan, bu esa mavjud adabiyotlarda keng yoritilmagan masalalardan biridir.

Asosiy qism

$\Omega = \{(x, t) : 0 < x < l, 0 < t < T\}$ sohada

$$\frac{\partial}{\partial t} \left(t^{2\gamma} \frac{\partial u}{\partial t} \right) - a^2 t^{2\gamma} \frac{\partial^2 u}{\partial x^2} = 0, \quad 0 < \gamma < 1/2 \quad (1)$$

tenglamani qaraylik.

A-masala. Ω sohada (1) tenglamani qanoatlantiruvchi va

$$u(x, 0) = f_1(x), \quad \lim_{t \rightarrow 0} t^{2\gamma} \left(t^{-2\gamma} (t^{2\gamma} u_t(x, t))'_t \right)' = f_2(x), \quad 0 \leq x \leq l; \quad (2)$$

boshlang'ich shartlarni hamda

$$u(0, t) = 0, \quad u(l, t) = 0, \quad 0 \leq t \leq T \quad (3)$$

chegaraviy shartlarni bajaruvchi $u(x, t) \in C(\bar{\Omega})$ funksiya topilsin, bu yerda $f_1(x)$ va $f_2(x)$ berilgan funksiyalar.

1-teorema. Agar A-masalaning yechimi mavjud bo'lsa, u yagonadir.

Isbot. Teskarisidan faraz qilaylik. A-masala ikkita $u_1(x, t)$ va $u_2(x, t)$ yechimlarga ega bo'lsin. Bu yechimlarning ayirmasi bir jinsli (1) tenglamani va (2)-(3) shartlarga mos bir jinsli boshlang'ich va chegaraviy shartlarni qanoatlantiradi. Bu ayirmani $u(x, t)$ bilan belgilaymiz, ya'ni

$$u(x, t) = u_1(x, t) - u_2(x, t). \quad (4)$$

Ma'lumki,

$$X_k(x) = \sqrt{\frac{2}{l}} \sin \lambda_k x, \quad \lambda_k = \frac{k\pi}{l}, \quad k = 1, 2, \dots, \quad (5)$$

funksiya $L_2(0, l)$ fazoda to'la ortonormallangan sistemani tashkil etadi [8].

Quyidagi

$$d_k(t) = \int_0^l u(x, t) X_k(x) dx, \quad k = 1, 2, \dots \quad (6)$$

funksiyani qaraymiz. (6) ni quyidagi ko‘rinishda yozib olamiz:

$$d_{k,\varepsilon}(t) = \int_{\varepsilon}^{l-\varepsilon} u(x,t) X_k(x) dx, \quad (7)$$

bunda $(\varepsilon, l-\varepsilon) \neq \emptyset$.

(7) tenglikni t bo‘yicha bir va ikki marta differensiallab quyidagi natijani olamiz

$$d'_{k,\varepsilon}(t) = \int_{\varepsilon}^{l-\varepsilon} u_t(x,t) X_k(x) dx, \quad d''_{k,\varepsilon}(t) = \int_{\varepsilon}^{l-\varepsilon} u_{tt}(x,t) X_k(x) dx.$$

(1) tenglamani e‘tiborga olib

$$d''_{k,\varepsilon}(t) + \frac{2\gamma}{t} d'_{k,\varepsilon}(t) = a^2 \int_{\varepsilon}^{l-\varepsilon} u_{xx}(x,t) X_k(x) dx. \quad (8)$$

(8) tenglikni hosil qilamiz. (8) tenglikning o‘ng tomonini ikki marta bo‘laklab integrallab, so‘ng $\varepsilon \rightarrow 0$ da limitga o‘tamiz.

$$\begin{aligned} \lim_{\varepsilon \rightarrow 0} \left[d''_{k,\varepsilon}(t) + \frac{2\gamma}{t} d'_{k,\varepsilon}(t) \right] &= a^2 \lim_{\varepsilon \rightarrow 0} \left[X_k(l-\varepsilon) u_x(l-\varepsilon, t) - X_k(\varepsilon) u_x(\varepsilon, t) \right] - \\ &- a^2 \sqrt{\lambda_k} \lim_{\varepsilon \rightarrow 0} \left[X'_k(l-\varepsilon) u(l-\varepsilon, t) - X'_k(\varepsilon) u(\varepsilon, t) \right] - a^2 \lambda_k \lim_{\varepsilon \rightarrow 0} \int_{\varepsilon}^{l-\varepsilon} u(x,t) X_k(x) dx \end{aligned}$$

(3) ga mos bir jinsli shartlar hamda $X_k(x)$ funksiyaning xossalriga asosan quyidagi natija kelib chiqadi

$$d''_{k,\varepsilon}(t) + \frac{2\gamma}{t} d'_{k,\varepsilon}(t) = a^2 \lambda_k \int_{\varepsilon}^{l-\varepsilon} u(x,t) X_k(x) dx, \quad k = 1, 2, \dots$$

(6) ni e‘tiborga olib quyidagi oddiy differensial tenglamaga kelamiz:

$$d''_{k,\varepsilon}(t) + \frac{2\gamma}{t} d'_{k,\varepsilon}(t) + a^2 \lambda_k d_k(t) = 0, \quad k = 1, 2, \dots \quad (9)$$

(2) shartga mos bir jinsli shartdan (6) ga asosan

$$d_k(0) = 0, \quad \lim_{t \rightarrow 0} t^{2\gamma} \left(t^{-2\gamma} \left(t^{2\gamma} d_k(x,t) \right)' \right)'_t = 0, \quad k = 1, 2, \dots, \quad (10)$$

boshlang‘ich shartlarni hosil qilamiz.

Ma‘lumki, (9) oddiy differensial tenglamaning umumiy yechimi quyidagi ko‘rinishda aniqlanadi [9]:

$$d_k(t) = a_k t^{1/2-\gamma} J_{1/2-\gamma}(\sqrt{\lambda_k} at) + b_k t^{1/2-\gamma} J_{\gamma-1/2}(\sqrt{\lambda_k} at).$$

Bu yechimni (10) shartlarga bo‘ysundirib $a_k = b_k = 0$ ga ega bo‘lamiz, ya‘ni barcha $\forall t \in [0, T]$ uchun $d_k(t) = 0$. U holda (6) tenglik quyidagi ko‘rinishni oladi:

$$\int_0^l u(x,t) X_k(x) dx = 0, \quad k = 1, 2, \dots$$

Demak, $u(x,t)$ funksiya (5) to‘la sistemadagi barcha funksiyalar bilan ortogonal ekan. Bu esa Ω sohaning deyarli barcha nuqtalarida $u(x,t) = 0$ ekanligini bildiradi. $u(x,t) \in C(\bar{\Omega})$ ekanligidan $u(x,t)$ funksiya $\bar{\Omega}$ sohaning barcha nuqtalarida $u(x,t) \equiv 0$ ekanligi kelib chiqadi. Bundan ko‘rinadiki, A- masalaning yechimi mavjud bo‘lsa u yagonadir. 1-teorema isbotlandi.

A masala yechimining mavjudligi.

2-teorema: Agar $f_1(x) \in C^4[0, l] \cap C^5(0, l)$, $f_1^{(5)}(x) \in L_2(0, l)$,
 $f_2(x) \in C[0, l] \cap C^1(0, l)$, $f_1'(x) \in L_2(0, l)$, $f_1(0) = f_1(l) = 0$, $f_1''(0) = f_1''(l) = 0$,
 $f_1^{(4)}(0) = f_1^{(4)}(l) = 0$, $f_2(0) = f_2(l) = 0$ shartlar bajarilsa, qo'yilgan masalaning yechimi mavjud.

Isbot. Bu masalani yechish uchun o'zgaruvchilarni ajratish (yoki Furiye) usuli qo'llaymiz. Avvalo (1) tenglamaning (3) shartlarini bajaruvchi yechimini

$$u(x, t) = X(x)T(t) \quad (14)$$

ko'rinishda izlaymiz, bu yerda $X(x)$ va $T(t)$ noma'lum funksiyalar. (14) ifodani (1) tenglamaga qo'yib, $X(x)$ va $T(t)$ noma'lum funksiyalarga nisbatan

$$T''(t) + \frac{2\gamma}{t}T'(t) + a^2\lambda T(t) = 0, \quad X''(x) + \lambda X(x) = 0 \quad (15)$$

tenglamalarni hosil qilamiz, bu yerda $\lambda = const$. (14) ifodadan va (3) chegaraviy shartlardan

$$X(0) = 0, \quad X(l) = 0 \quad (16)$$

chegaraviy shartlar kelib chiqadi.

(15)-(16) masala xos son va xos funksiyalarni topish haqidagi Shturm-Liuville masalasidir. (15)-(16) masalaning xos sonlari

$$\lambda_k = \left(\frac{\pi k}{l}\right)^2 \quad (k = 1, 2, \dots) \quad (17)$$

bu xos sonlarga mos trivial bo'lmagan (aynan nolga teng bo'lmagan) normallangan xos funksiyalari

$$X_k(x) = \sqrt{\frac{2}{l}} \sin \frac{\pi k}{l} x, \quad k \in N \quad (18)$$

bo'ladi. $\lambda = \lambda_k$ bo'lganda (1) tenglamaning yechimi

$$u(x, t) = \sum_{k=1}^{+\infty} T_k(t) \sqrt{\frac{2}{l}} \sin \frac{\pi k}{l} x, \quad (n = 1, 2, \dots), \quad (19)$$

ko'rinishga qidiramiz, bu yerda $T_k(t)$, $k \in N$ - hozircha noma'lum funksiya.

(19) funksiyani (1) tenglamaga qo'yib

$$\sum_{k=1}^{+\infty} \left[T_k''(t) + \frac{2\gamma}{t} T_k'(t) + a^2 \lambda_k T_k(t) \right] \sqrt{\frac{2}{l}} \sin \frac{\pi k}{l} x = 0,$$

tenglikni hosil qilamiz. Bu tenglikdan

$$T_k''(t) + \frac{2\gamma}{t} T_k'(t) + a^2 \lambda_k T_k(t) = 0, \quad (20)$$

ikkinchi tartibli oddiy differensial tenglamani hosil qilamiz.

(20) tenglamada

$$a^2 \lambda_k = \mu_k \quad \text{va} \quad T_k(t) = \left(\frac{z}{\sqrt{\mu_k}} \right)^{1/2-\gamma} g_k(z), \quad \text{bu yerda} \quad z = \sqrt{\mu_k} t$$

ko'rinishida belgilash kiritib

$$g_k''(z) + \frac{1}{z} g_k'(z) + \left[1 - \frac{(1/2 - \gamma)^2}{z^2} \right] g_k(z) = 0$$

1-tur Bessel tenglamasiga keltiramiz. Ma'lumki tenglamaning umumiy yechimi

$$g_k(z) = a_k J_{1/2-\gamma}(z) + b_k J_{\gamma-1/2}(z)$$

ko'rinishda aniqlanadi. Ohirgi umumiy yechimdan $T_k(t) = (z/\sqrt{\mu_k})^{1/2-\gamma} g_k(z)$, $z = \sqrt{\mu_k} t$ almashtirishga asosan ortga qaytib, (20) tenglamaning umumiy yechimini quyidagicha topamiz:

$$T_k(t) = a_k t^{1/2-\gamma} J_{1/2-\gamma}(\sqrt{\mu_k} t) + b_k t^{1/2-\gamma} J_{\gamma-1/2}(\sqrt{\mu_k} t). \quad (21)$$

(21) tenglikni (19) tenglikka qo'yib

$$u(x, t) = \sum_{k=1}^{\infty} \left[a_k t^{1/2-\gamma} J_{1/2-\gamma}(\sqrt{\mu_k} t) + b_k t^{1/2-\gamma} J_{\gamma-1/2}(\sqrt{\mu_k} t) \right] X_k(x) \quad (22)$$

qo'yilgan masalaning (3) shartlarini qanoatlantiruvchi yechimini hosil qilamiz, bu yerda a_k va b_k Furye koeffitsiyenti, (2) boshlang'ich shartlardan

$$a_k = -\frac{1}{8} \left(\frac{\sqrt{\mu_k}}{2} \right)^{\gamma-5/2} \Gamma\left(\frac{1}{2} - \gamma\right) \int_0^l f_2(x) \sqrt{\frac{2}{l}} \sin \frac{\pi k}{l} x dx, \quad (23)$$

$$b_k = \left(\frac{\sqrt{\mu_k}}{2} \right)^{1/2-\gamma} \Gamma\left(\frac{1}{2} + \gamma\right) \int_0^l f_1(x) \sqrt{\frac{2}{l}} \sin \frac{\pi k}{l} x dx \quad (24)$$

kelib chiqadi.

Shunday qilib qo'yilgan masalaning yechimi (22) ko'rinishda aniqlanadi, bu yerda a_k va b_k lar (23) va (24) tengliklardan $X_k(x)$ esa mos ravishda (18) tenglik yordamida aniqlanadi.

Lemma. $\forall t \in [0, T]$ va $k = 1, 2, \dots$ bo'lganda quyidagi baholar o'rinli bo'ladi:

$$|T_k(t)| \leq |f_{1k}| + C_1 \mu_k^{-1} |f_{2k}|, \quad (25)$$

$$\left| t^{-2\gamma} \frac{\partial}{\partial t} [t^{2\gamma} T_k'(t)] \right| \leq \mu_k |f_{1k}| + C_1 |f_{2k}|, \quad (26)$$

$$\left| t^{2\gamma} \left(t^{-2\gamma} (t^{2\gamma} T_k'(t))' \right)' \right| \leq C_2 \mu_k^2 |f_{1k}| + |f_{2k}|, \quad (27)$$

bu yerda

$$f_{1k} = \int_0^l f_1(x) X_k(x) dx, \quad f_{2k} = \int_0^l f_2(x) X_k(x) dx, \quad (28)$$

Furye koeffitsiyentlari, C_1 va C_2 - musbat o'zgarmas sonlar.

Isbot. $\bar{J}_\nu(z) = \Gamma(\nu + 1) \times (z/2)^{-\nu} J_\nu(z)$ ya'ni Bessel-Klifford funksiyasi yordamida, (21) tenglikni quyidagicha yozib olamiz:

$$T_k(t) = a_k t^{1/2-\gamma} \left(\frac{\sqrt{\mu_k} t}{2} \right)^{1/2-\gamma} \frac{\bar{J}_{1/2-\gamma}(\sqrt{\mu_k} t)}{\Gamma(1/2-\gamma)(1/2-\gamma)} + b_k t^{1/2-\gamma} \left(\frac{\sqrt{\mu_k} t}{2} \right)^{\gamma-1/2} \frac{\bar{J}_{\gamma-1/2}(\sqrt{\mu_k} t)}{\Gamma(\gamma+1/2)}.$$

($\nu > -1/2$) bo'lganda $|\bar{J}_\nu(z)| \leq 1$ tengsizlik o'rinli bo'ladi []. Buni e'tiborga olib oxirgi tenglikni quyidagicha baholaymiz:

$$\begin{aligned} |T_k(t)| &\leq \left| a_k t^{1/2-\gamma} \left(\frac{\sqrt{\mu_k t}}{2} \right)^{1/2-\gamma} \frac{\bar{J}_{1/2-\gamma}(\sqrt{\mu_k t})}{\Gamma(1/2-\gamma)(1/2-\gamma)} \right| + \left| b_k t^{1/2-\gamma} \left(\frac{\sqrt{\mu_k t}}{2} \right)^{\gamma-1/2} \frac{\bar{J}_{\gamma-1/2}(\sqrt{\mu_k t})}{\Gamma(\gamma+1/2)} \right| \leq \\ &\leq \left| a_k t^{1/2-\gamma} \left(\frac{\sqrt{\mu_k t}}{2} \right)^{1/2-\gamma} \frac{1}{\Gamma(1/2-\gamma)(1/2-\gamma)} \right| + \left| b_k t^{1/2-\gamma} \left(\frac{\sqrt{\mu_k t}}{2} \right)^{\gamma-1/2} \frac{1}{\Gamma(\gamma+1/2)} \right|. \end{aligned}$$

Hosil bo'lgan tengsizlikdagi a_k va b_k koeffitsiyentlarning o'rniga mos holda (23) va (24) ko'rinishlarini qo'yib soddalashtirsak, (25) tengsizlikni hosil qilamiz. Shu tarzda (26) va (27) tengsizliklarni ham ko'rsatish mumkin.

Endi (19) yechimni yaqinlashuvchi ekanligini tekshiramiz. Buning uchun unga majorant qator tuzamiz.

$$|u(x, t)| \leq \sum_{k=1}^{+\infty} |T_k(t)| \sqrt{\frac{2}{l}} \left| \sin \frac{\pi k}{l} x \right| \leq \sum_{k=1}^{+\infty} |T_k(t)| \sqrt{\frac{2}{l}},$$

bu funksional qatorga (25) tengsizlikni qo'yib

$$\sum_{k=1}^{+\infty} [|f_{1k}| + C_1 \mu_k^{-1} |f_{2k}|] \sqrt{\frac{2}{l}},$$

sonli qatorni hosil qilamiz.

(28) integrallarning birinchisini bir marta bo'laklab integrallab $f_1(x)$ funksiyaga teoramada qo'yilgan shartlarni inobatga olsak

$$\sum_{k=1}^{+\infty} \left[\frac{l}{\pi k} |f_{1k}^{(1)}| + C_1 \frac{l^2}{a^2 (\pi k)^2} |f_{2k}| \right] \sqrt{\frac{2}{l}}$$

qator hosil bo'ladi, bu yerda $f_{1k}^{(1)} = \int_0^l f_1'(x) \sqrt{\frac{2}{l}} \cos \frac{\pi k}{l} x dx$. Hosil bo'lgan qatordan o'zgarmas sonni tashlab yuborib

$$\sum_{k=1}^{+\infty} \left[\frac{1}{k} |f_{1k}^{(1)}| + \frac{1}{k^2} |f_{2k}| \right],$$

qatorni hosil qilamiz. Bu hosil bo'lgan qator yaqinlashuvchi, chunki $X_k(x) = \sqrt{\frac{2}{l}} \sin \frac{\pi k}{l} x$ funksiyalar $(0, l)$ oraliqda ortonormal sistemanini tashkil etadi.

Shuning uchun har qanday $f \in L_2(0, l)$ uchun Parseval tengligi bajariladi:

$$\sum_{k=1}^{+\infty} |f_k|^2 = \int_0^l f^2(x) dx = \|f_k\|_{L_2(0, l)}^2, \quad f_k = \int_0^l f(x) X_k(x) dx.$$

Bundan tashqari, Koshi–Bunyakovskiy tengsizligidan foydalanib,

$$\sum_{k=1}^{+\infty} |a_k b_k| \leq \left(\sum_{k=1}^{+\infty} a_k^2 \right)^{1/2} \left(\sum_{k=1}^{+\infty} b_k^2 \right)^{1/2} = \|a_k\|_{L_2(0, l)} \|b_k\|_{L_2(0, l)},$$

bizning qatorlar uchun

$$\sum_{k=1}^{+\infty} \frac{1}{k} |f_{1k}^{(1)}| \leq \left(\sum_{k=1}^{+\infty} \frac{1}{k^2} \right)^{1/2} \left(\sum_{k=1}^{+\infty} |f_{1k}|^2 \right)^{1/2} = \frac{\pi}{\sqrt{6}} \|f_{1k}^{(1)}\|_{L_2(0,l)},$$

$$\sum_{k=1}^{+\infty} \frac{1}{k^2} |f_{2k}| \leq \left(\sum_{k=1}^{+\infty} \frac{1}{k^4} \right)^{1/2} \left(\sum_{k=1}^{+\infty} |f_{2k}|^2 \right)^{1/2} = \frac{\pi^2}{\sqrt{90}} \|f_{2k}\|_{L_2(0,l)}.$$

Shu sababli, Parseval tengligi va Koshi–Bunyakovskiy tengsizligi natijasida

$$\sum_{k=1}^{+\infty} \left(\frac{1}{k} |f_{1k}| + \frac{1}{k^2} |f_{2k}| \right) < \infty,$$

ya'ni qator yaqinlashuvchidir. Bundan esa (19) funksional qator $\bar{\Omega}$ sohada tekis va absolyut yaqinlashuvchiligi kelib chiqadi. Yuqoridagi jarayonni takrorlab

$$\left| t^{2\gamma} \left(t^{-2\gamma} \left(t^{2\gamma} u_t(x, t) \right)' \right)' \right|_t = \sum_{k=1}^{+\infty} \left| t^{2\gamma} \left(t^{-2\gamma} \left(t^{2\gamma} T_k'(t) \right)' \right)' \right| |X_k(x)| \quad (29)$$

funksional qatorni tekis yaqinlashuvchilikka tekshiramiz. Buning uchun (29) ga (27) tengsizlikni qo'yib, $|X_k(x)| \leq 1$ tengsizlikni e'tiborga olib

$$\sum_{k=1}^{+\infty} [C_2 \mu_k^2 |f_{1k}| + |f_{2k}|]$$

sonli qatorni hosil qilamiz.

(28) integrallarning birinchisini besh marta, ikkinchisini esa bir marta bo'laklab integrallab $f_1(x)$ va $f_2(x)$ funksiyaga teoramada qo'yilgan shartlarni inobatga olib o'zgarmaslarni tashlab yuborib

$$\sum_{k=1}^{+\infty} \left[\frac{1}{k} |f_{1k}^{(5)}| + \frac{1}{k} |f_{2k}^{(1)}| \right]$$

qator hosil bo'ladi, bu yerda $f_{1k}^{(5)} = \int_0^l f_1^{(5)}(x) \sqrt{\frac{2}{l}} \cos \frac{\pi k}{l} x dx$, $f_{2k}^{(1)} = \int_0^l f_2'(x) \sqrt{\frac{2}{l}} \cos \frac{\pi k}{l} x dx$.

Bu hosil bo'lgan qator Parseval tengligi va Koshi–Bunyakovskiy tengsizligiga asosan yaqinlashuvchi. Bundan esa $t^{2\gamma} \left(t^{-2\gamma} \left(t^{2\gamma} u_t(x, t) \right)' \right)'$ ifodaga mos keladigan funksional qator $\bar{\Omega}$ sohada tekis va absolyut yaqinlashuvchiligi kelib chiqadi.

Xulosa

Yuqorida isbotlanganlardan quyidagilar kelib chiqadi:

har bir qator, (1)-tenglamaning hadlariga mos keluvchi funksional qatorlar $K \in \Omega$ kompakt to'plamda absolyut va tekis yaqinlashadi,

(22) funksional qator (1)-tenglamani qanoatlantiradi va (2), (3)-shartlarni bajaradi.

Demak, (22) funksional qator qo'yilgan masalaning yechimi yechimi bo'ladi.

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Переходные процессы в рельефных участках трубопроводов для транспортировки несжимаемой жидкости, образованные изменением скорости в концах рельефного участка

Хужаев И.К.¹, Хожиқулов Ш.Ш.¹, Махкамов М.К.² Нуманжанов А.А.².

¹ Механики и сейсмостойкости сооружений им. М.Т. Уразбаева АН РУз, Ташкент, Узбекистан,

² Андижанский государственный университет, Узбекистан, г. Андижан

2025 йил. 2-июлда берилган; 2025 йил. 26-ноябрда чоп этиш учун қабул қилинган

Аннотация. В рамках не консервативной формы квазиодномерных линеаризованных уравнений Н.Е. Жуковского сформулирована задача о переходном процессе на элементарном рельефном участке трубопровода при изменении граничных скоростей несжимаемой жидкости по времени. Неполное телеграфное уравнение, составленное для осредненной скорости потока, решено методом разделения переменных. Значение давления получено интегрированием исходных уравнений с привлечением полученного решения по скорости.

Представлены численные результаты, относящиеся случаю постоянных значений функций в начальных и граничных условиях. Выделены варианты постоянного, возрастающего и убывающего среднего по участку значения давления. Выявлены особенности поведения ударных волн с учетом и без учета силы трения.

Ключевые слова: трубопровод, несжимаемая жидкость, скорость, давления, сила сопротивления, гравитация, квазиодномерная модель, метод Фурье, вычислительный эксперимент.

Аннотация: N.E. Jukovskiyning kvazi bir o'lchovli chiziqli tenglamalarining konservativ bo'lmagan shakli doirasida, siqilmaydigan suyuqlikning chegara tezligining vaqt o'tishi bilan o'zgarishi hamda quvur uchastkasining elementar relyef qismidagi vaqtinchalik jarayon muammosi ishlab chiqilgan. O'rtacha oqim tezligi uchun tuzilgan to'liq bo'lmagan telegraf tenglamasi o'zgaruvchilarni ajratish usuli bilan hal qilinadi. Bosim qiymati tezlik bo'yicha olingan dastlabki tenglamalarni integrallash yo'li bilan olinadi.

Soni natijalar boshlang'ich va chegara shartlaridan funktsiyalarning doimiy qiymatlari holati bilan bog'liq. Kesimdagi o'rtacha bosimning doimiy, ortib borayotgan va kamayuvchi qiymatlari variantlari aniqlanadi. Ishqalanish kuchini hisobga olgan holda va hisobga olinmagan zarba to'lqinlarining xatti-harakatining xususiyatlari aniqlanilgan.

Калит сўзлар: quvur, siqilmaydigan suyuqlik, tezlik, bosim, qarshilik kuchi, tortishish, kvazi bir o'lchovli model, Furey usuli, hisoblash tajribasi.

Abstract: Within the framework of the non-conservative form of quasi-one-dimensional linearized equations of N.E. Zhukovsky, the problem of the transient process on an elementary relief section of a pipeline with a change in the boundary velocities of an incompressible fluid over time is formulated. The incomplete telegraph equation, compiled for the average flow velocity, is solved by the method of separation of variables. The pressure value is obtained by integrating the original equations using the obtained solution for the velocity. Numerical results are presented related to the case of constant values of functions in the initial and boundary conditions. Variants of constant, increasing and decreasing average pressure values over the section are identified. Features of the behavior of shock waves with and without taking into account the friction force are revealed.

Keywords: pipeline, slightly compressible fluid, velocity, pressure, resistance force, gravity, quasi-one-dimensional model, Fure method, computational experiment..

I. Введение

Сформулирована и решена нестационарная задача о состоянии мало-сжимаемой жидкости, когда изменение плотности жидкости и деформируемость тонкой стенки учитываются при формировании скорости распространения малых возмущений давления по Н.Е. Жуковскому, а начальные распределения скорости и давления известны. Путевое изменение давления обусловлено не консервативной формой локальной составляющей силы инерции и силой сопротивления, которая линеаризована. Временное изменение давления обусловлено градиентом скорости по расстоянию.

В инженерной практике гидравлический расчет трубопроводов и их сетей, как правило, осуществляется на основе характеристик стационарного потока, удалённого от зон возмущений [1]. Однако при рассмотрении нестационарных процессов выявляются так называемые «узкие места» трубопроводной системы. Типичным примером нестационарной задачи является задача переходных процессов. Переход от одного установившегося режима к другому, как правило, сопровождается явлениями, требующими одновременного учёта двух характерных масштабов скорости: гидродинамической скорости потока и скорости распространения малых возмущений в системе

«труба – транспортируемая среда». Образование и распространение волн возмущения в трубопроводах изучаются в рамках квазиодномерных уравнений Н.Е. Жуковского [2,3]. Им впервые построена система квазиодномерных уравнений, которые одновременно учитывают гидродинамической скорости текучей среды и скорости распространения малых возмущений в системе среда–трубопровод, и проведены теоретические и экспериментальные исследования по распространению волн уплотнения и разряжения в трубопроводах.

Задачи о распространении волн уплотнения и разряжения представляют особый интерес для науки и практики. Многократные повторения таких волн приводит к потере целостности трубопровода. Приведем два примера, которые демонстрируют разрушительные последствия повторений чередования волн уплотнения и разряжения.

Пуск и остановка агрегатов насосных станций сопровождается образованием волн возмущений давления в больших, разрушительных интервалах [4].

В целом, изучении особенностей образования и распространения ударных волн представляет большой интерес. Трудность процесса изучения ударных волн связана со скачками показателей. Это относится и к теоретическим исследованиям, и стендовым исследованиям. В первом случае это обусловлено использованием функции Хэвисайда [5] или же введением бегущих волн и разработкой адекватных численных методов [6,7]. Во втором случае это связано разработкой и использованием менее инерционных систем измерения с поддержкой современных компьютерных технологий [8,9].

Исходя из этих особенностей ударной волны и потребностей инженерной практики к исследованиям, ниже рассматривается элементарный рельефный участок трубопровода, в обоих концах которого заданы законы изменения скорости по времени. Начальные распределения скорости и ее производной по времени известны. Требуется решить задачу методом Фурье в рамках квазиодномерной модели Н.Е. Жуковского.

Предлагается общий вариант решения задачи с учетом силовых факторов по трению, гравитации и локальной составляющей силы инерции при переменных входных и выходных

скоростях. Численные результаты получены для постоянных значений функций, фигурирующих в начальных и граничных условиях. В отличие от известных решений здесь учитывается рельеф трассы прокладки трубопровода.

Постановка и метод решения задачи

Динамическое состояние жидкости в элементарном участке трубопровода при $\rho \approx const$ описывается уравнениями [2,10]:

$$\begin{cases} -\frac{\partial P}{\partial x} = \rho \left(\frac{\partial w}{\partial t} + 2aw \right), \\ -\frac{\partial P}{\partial t} = \rho c^2 \frac{\partial w}{\partial x}. \end{cases} \quad (1)$$

Здесь $P(x,t) = p(x,t) + \rho gy(x)$ – полное гидростатическое давление, представляющее меру потери энергии жидкости; $p(x,t)$, $w(x,t)$ – средние по сечению x значения давления и скорости потока в момент времени t ; $y(x)$ – нивелирная высота оси трубопровода; $a = \frac{\lambda w_*}{4D}$;

w_* – характерная скорость процесса – параметр осреднения; λ – коэффициент сопротивления трения; $c = \left(\frac{\rho_0}{k} + \frac{D\rho_0}{E\delta} \right)^{-1/2}$ – скорость

распространения малых возмущений давления в системе труба-жидкость; ρ_0 , D – плотность жидкости и диаметр трубопровода в невозмущенном состоянии; k , E – модули упругости транспортируемой жидкости и трубы; δ – толщина трубы [10].

Исключение полного напора $P(x,t)$ при постоянном значении площади поперечного сечения трубы $f = \pi D^2 / 4$ позволяет построить отдельное уравнение относительно скорости:

$$\frac{\partial^2 w}{\partial t^2} + 2a \frac{\partial w}{\partial t} = c^2 \frac{\partial^2 w}{\partial x^2}.$$

Это – неполное телеграфное уравнение [17,16].

Рассматривается вариант начальных условий:

$$w(x,0) = w^{(0)}(x), \quad \frac{\partial w(x,0)}{\partial t} = w^{(1)}(x).$$

Граничные условия заданы в виде

$$w(0,t) = \bar{w}(t), \quad w(l,t) = \tilde{w}(t).$$

Согласно этим данным и первому уравнению исходной системы начальным условием для полного напора служит

$$P(x,0) = p_{00} - \rho \int_0^x [w^{(t)}(\eta) + 2aw^{(0)}(\eta)] d\eta,$$

где p_{00} – начальное давление во входном сечении.

Если же второе условие задано в виде $P(x,0) = p_0(x)$, то с привлечением первого уравнения исходной системы его можно преобразовать, т.е. два вида второго условия приводят к одинаковому результату.

Приведем граничные условия к однородному виду. Для этой цели вводится новая искомая функция $u(x,t)$ согласно зависимости

$$w(x,t) = C_0(t) + xC_1(t) + u(x,t).$$

Определены значения функций

$$C_0(t) = \bar{w}(t), \quad C_1(t) = \frac{\tilde{w}(t) - \bar{w}(t)}{l},$$

которые позволяют представить граничные условия в однородном виде:

$$u(0,t) = 0, \quad u(l,t) = 0.$$

При этом само уравнение становится неоднородным

$$\frac{\partial^2 u}{\partial t^2} + 2a \frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} - H(t) - xR(t).$$

Оно решается при новых начальных условиях

$$u(x,0) = w^{(0)}(x) - C_0(0) - C_1(0)x,$$

$$\frac{\partial u(x,0)}{\partial t} = w^{(t)}(x) - C'_0(0) - C'_1(0)x$$

и при известных уже однородных граничных условиях.

Здесь и далее пользуемся обозначениями

$$H(t) = C''_0(t) + 2aC'_0(t), \quad R(t) = C''_1(t) + 2aC'_1(t).$$

Приступим к решению уравнения относительно вспомогательной функции. Уравнения и краевые условия являются линейными. Это облегчает процесс решения задачи, и решение ищем в виде суммы суперпозиции двух решений [10,11].

Первое слагаемое $U(x,t)$ является общим решением однородного уравнения

$$\frac{\partial^2 U}{\partial t^2} + 2a \frac{\partial U}{\partial t} = c^2 \frac{\partial^2 U}{\partial x^2}$$

при заданных граничных $U(0,t) = 0, U(l,t) = 0$ и начальных условиях

$$U(x,0) = w^{(0)}(x) - C_0(0) - C_1(0)x,$$

$$\frac{\partial U(x,0)}{\partial t} = w^{(t)}(x) - C'_0(0) - C'_1(0)x.$$

Второе слагаемое $V(x,t)$ представляет частное решение неоднородного уравнения в соответствии с правой частью

$$\frac{\partial^2 V}{\partial t^2} + 2a \frac{\partial V}{\partial t} = c^2 \frac{\partial^2 V}{\partial x^2} - H(t) - xR(t),$$

которое решается при нулевых начальных

$$V(x,0) = 0, \quad \frac{\partial V(x,0)}{\partial t} = 0.$$

и однородных граничных условиях

$$V(0,t) = 0, \quad V(l,t) = 0.$$

Сначала решается однородное уравнение.

Положим, что

$$U(x,t) = X(x)Y(t),$$

и приходим к уравнениям:

$$\frac{Y''(t) + 2aY'(t)}{c^2Y(t)} = \frac{X''(x)}{X(x)} = -\lambda^2.$$

Задача Штурма-Лиувилля по координате x дает [10,11,12]

$$X_n(x) = \sin \lambda_n x, \quad \lambda_n = \frac{n\pi}{l}, \quad \|X_n(x)\|^2 = \frac{l}{2}.$$

При этом для определения множителя в виде собственной функции от времени имеем уравнение

$$Y_n''(t) + 2aY_n'(t) + c^2\lambda_n^2 Y_n(t) = 0. \quad (2)$$

С учетом этих возможных вариантов $D_n > 0$, $D_n = 0$ и $D_n < 0$ для $Y_n(t)$ получим решение:

$$Y_n(t) = \begin{cases} e^{-at} (A_n \operatorname{ch} \sqrt{D_n} t + B_n \operatorname{sh} \sqrt{D_n} t) & \text{при } D_n > 0, \\ e^{-at} (A_n + B_n t) & \text{при } D_n = 0, \\ e^{-at} (A_n \cos \sqrt{|D_n|} t + B_n \sin \sqrt{|D_n|} t) & \text{при } D_n < 0. \end{cases}$$

Соответственно, общее решение однородного уравнения составляет

$$U(x,t) = \sum_{n=1}^{\infty} Y_n(t) \sin \lambda_n x.$$

Переходим к реализации граничных условий общего решения однородного уравнения.

Из первого начального условия (при $t = 0$) следует:

$$\sum_{n=1}^{\infty} A_n \sin \lambda_n x = w^{(0)}(x) - C_0(0) - xC_1(0).$$

Умножим сторон этого уравнения на $\sin \lambda_n x dx$ и интегрируем от нуля до l . В результате получим

$$A_n \|X_n(x)\|^2 = W_n - C_0(0)S_n - C_1(0)K_n,$$

где

$$W_n = \int_0^l w^{(0)}(x) \sin \lambda_n x dx,$$

$$S_n = \int_0^l \sin \lambda_n x dx = \frac{l}{n\pi} [1 - (-1)^n],$$

$$K_n = \int_0^l x \sin \lambda_n x dx = \frac{l^2}{n\pi} (-1)^{n+1}.$$

Отсюда определим

$$A_n = \frac{2}{l} [W_n - C_0(0)S_n - C_1(0)K_n].$$

Второе начальное условие связано с производной от функции $U(x, t)$ по времени при $t = 0$.

Находим

$$Y'_n(t) = \begin{cases} e^{-at} \left[\begin{aligned} &(-aB_n + \sqrt{D_n} A_n) sh \sqrt{D_n} t + \\ &+ (-aA_n + \sqrt{D_n} B_n) ch \sqrt{D_n} t \end{aligned} \right] & \text{при } D_n > 0, \\ e^{-at} [-a(A_n + B_n t) + B_n] & \text{при } D_n = 0, \\ e^{-at} \left[\begin{aligned} &(-aB_n - \sqrt{|D_n|} A_n) \sin \sqrt{|D_n|} t + \\ &+ (-aA_n + \sqrt{|D_n|} B_n) \cos \sqrt{|D_n|} t \end{aligned} \right] & \text{при } D_n < 0. \end{cases}$$

Вычислим

$$Y'_n(0) = \begin{cases} -aA_n + \sqrt{D_n} B_n & \text{при } D_n > 0, \\ -aA_n + B_n & \text{при } D_n = 0, \\ -aA_n + \sqrt{|D_n|} B_n & \text{при } D_n < 0 \end{cases} = -aA_n + \gamma_n B_n,$$

$$\text{где } \gamma_n = \begin{cases} \sqrt{D_n} & \text{при } D_n > 0, \\ 1 & \text{при } D_n = 0, \\ \sqrt{|D_n|} & \text{при } D_n < 0. \end{cases}$$

Тогда второе начальное условие приобретает вид:

$$\sum_{n=1}^{\infty} (-aA_n + \gamma_n B_n) \sin \lambda_n x = w^{(t)}(x) - C'_0(0) - xC'_1(0).$$

Применение ортонормированности собственных функций по x , при

$$W_n^{(t)} = \int_0^l w^{(t)}(\eta) \sin \lambda_n \eta d\eta, \text{ приводит к равенству}$$

$$(-aA_n + \gamma_n B_n) \frac{l}{2} = W_n^{(t)} - C'_0(0)S_n - C'_1(0)K_n.$$

Отсюда находим значения коэффициентов

$$B_n = \frac{a}{\gamma_n} A_n + \frac{2}{\gamma_n l} [W_n^{(t)} - C'_0(0)S_n - C'_1(0)K_n].$$

Определены значения коэффициентов общего решения однородного уравнения. Теперь переходим к решению неоднородного уравнения.

Полагаем, что

$$V(x, t) = \sum_{n=1}^{\infty} L_n(t) \sin \lambda_n x,$$

тем выполняем граничные условия $V(0, t) = 0, V(l, t) = 0$.

Подстановка данного решения в неоднородное уравнение приводит к уравнению

$$\sum_{n=1}^{\infty} [L''_n(t) + 2aL'_n(t) + c^2 \lambda_n^2 L_n(t)] \sin \lambda_n x = -H(t) - xR(t).$$

Используем условия ортонормированности собственных функций $X_n(x) = \sin \lambda_n x$:

$$L''_n(t) + 2aL'_n(t) + c^2 \lambda_n^2 L_n(t) = \frac{2}{l} [-H(t)S_n - R(t)K_n]. \quad (3)$$

При $t = 0$ имеем условие $L_n(0) = 0$. Из второго начального условия $\frac{\partial V(x, 0)}{\partial t} = 0$ следует, что $L'_n(0) = 0$.

В целом, рассматриваются случаи, когда функции $L_n(t)$ определены аналитически. Такому классу относятся случаи линейной, экспоненциальной, тригонометрической и некоторых других форм задания граничных условий для скорости. При этом окончательное решение для вспомогательной функции $u(x, t)$ определяется в виде

$$u(x, t) = \sum_{n=1}^{\infty} [Y_n(t) + L_n(t)] \sin \lambda_n x.$$

Переходя обратно, получим решение задачи по скорости

$$w(x, t) = C_0(t) + xC_1(t) + \sum_{n=1}^{\infty} [Y_n(t) + L_n(t)] \sin \lambda_n x.$$

Решение задачи относительно давления находим, интегрируя исходную систему квазиодномерных уравнений.

В первом шаге решения $P(x, t)$ обратимся ко второму уравнению $\frac{\partial P}{\partial t} = -\rho c^2 \frac{\partial w}{\partial x}$ исходной системы.

Находим

$$\frac{\partial w(x, t)}{\partial x} = C_1(t) + \sum_{n=1}^{\infty} \lambda_n [Y_n(t) + L_n(t)] \cos \lambda_n x.$$

В результате интегрирования второго уравнения исходной системы получим формулу для давления:

$$P(x, t) = -\rho c^2 \left[C_1^{(0)}(t) + \sum_{n=1}^{\infty} \lambda_n [Y_n^{(0)}(t) + L_n^{(0)}(t)] \cos \lambda_n x \right] + p_t'(x).$$

Здесь

$$\int_0^t C_1(\theta) d\theta = C_1^{(0)}(t) \text{ и } \int_0^t L_n(\theta) d\theta = L_n^{(0)}(t),$$

$$Y_n^{(0)}(t) = \int_0^t Y_n(\theta) d\theta =$$

$$\begin{cases} \frac{1}{c^2 \lambda_n^2} \left[(-A_n a - B_n \sqrt{D_n}) (e^{-at} \operatorname{ch} \sqrt{D_n} t - 1) + \right. \\ \left. + (-A_n \sqrt{D_n} - B_n a) e^{-at} \operatorname{sh} \sqrt{D_n} t \right] & \text{при } D_n > 0, \\ A_n \frac{1 - e^{-at}}{a} + B_n \left(-\frac{te^{-at}}{a} + \frac{1 - e^{-at}}{a^2} \right) & \text{при } D_n = 0, \\ \frac{1}{c^2 \lambda_n^2} \left[(-A_n a - B_n \sqrt{|D_n|}) (e^{-at} \cos \sqrt{|D_n|} t - 1) + \right. \\ \left. + (A_n \sqrt{|D_n|} - B_n a) e^{-at} \sin \sqrt{|D_n|} t \right] & \text{при } D_n < 0. \end{cases}$$

При использовании решения для случая $a \rightarrow 0$ (когда сила трения пренебрежимо мала) в выражении $Y_n^{(0)}(t)$ необходимо учитывать значения пределов

$$e^{-at} \rightarrow 1, \quad \frac{1 - e^{-at}}{a} \rightarrow t, \quad -\frac{te^{-at}}{a} + \frac{1 - e^{-at}}{a^2} \rightarrow t^2$$

и принять

$$\lim_{a \rightarrow 0} \left[A_n \frac{1 - e^{-at}}{a} + B_n \left(-\frac{te^{-at}}{a} + \frac{1 - e^{-at}}{a^2} \right) \right] = A_n t + B_n t^2.$$

Здесь надо определить значение $p_t'(x)$ – постоянного интегрирования по времени. Для определения его обратимся к первому уравнению исходной системы (1).

Вычислим

$$\frac{\partial p(x, t)}{\partial x} = \rho c^2 \sum_{n=1}^{\infty} \lambda_n^2 [Y_n^{(0)}(t) + L_n^{(0)}(t)] \sin \lambda_n x + p_t'(x)$$

и

$$\frac{\partial w(x, t)}{\partial t} = C_0'(t) + x C_1'(t) + \sum_{n=1}^{\infty} [Y_n'(t) + L_n'(t)] \sin \lambda_n x.$$

Эти выражения и выражение для скорости $w(x, t)$ вставим в первое уравнение системы (1):

$$\begin{aligned} -\rho c^2 \sum_{n=1}^{\infty} \lambda_n^2 [Y_n^{(0)}(t) + L_n^{(0)}(t)] \sin \lambda_n x - p_t'(x) = \\ = \rho \left[C_0'(t) + x C_1'(t) + \sum_{n=1}^{\infty} [Y_n'(t) + L_n'(t)] \sin \lambda_n x \right] + \\ + 2a \rho \left[C_0(t) + x C_1(t) + \sum_{n=1}^{\infty} [Y_n(t) + L_n(t)] \sin \lambda_n x \right]. \end{aligned}$$

Отсюда находим

$$\begin{aligned} p_t'(x) = -\rho [C_0'(t) + 2a C_0(t)] - \rho [C_1'(t) + 2a C_1(t)] x - \\ - \rho \sum_{n=1}^{\infty} [Y_n'(t) + 2a Y_n(t) + c^2 \lambda_n^2 Y_n^{(0)}(t)] \sin \lambda_n x - \\ - \rho \sum_{n=1}^{\infty} [L_n'(t) + 2a L_n(t) + c^2 \lambda_n^2 L_n^{(0)}(t)] \sin \lambda_n x. \end{aligned}$$

Преобразуем первую сумму

$$\begin{aligned} \sum_{n=1}^{\infty} [Y_n'(t) + 2a Y_n(t) + c^2 \lambda_n^2 Y_n^{(0)}(t)] \sin \lambda_n x = \\ = \sum_{n=1}^{\infty} \int_0^t [Y_n''(\theta) + 2a Y_n'(\theta) + c^2 \lambda_n^2 Y_n(\theta)] d\theta \sin \lambda_n x = \\ = \sum_{n=1}^{\infty} c_n \sin \lambda_n x = E(x). \end{aligned}$$

Аналогично поступим со второй суммой:

$$\begin{aligned} \sum_{n=1}^{\infty} [L_n'(t) + 2a L_n(t) + c^2 \lambda_n^2 L_n^{(0)}(t)] \sin \lambda_n x = \\ = \sum_{n=1}^{\infty} \int_0^t [L_n''(\theta) + 2a L_n'(\theta) + c^2 \lambda_n^2 L_n(\theta)] d\theta \sin \lambda_n x = \\ = \int_0^t \frac{2}{l} [-H(\theta) S_n - R(\theta) K_n] d\theta = \frac{2}{l} [-H^{(0)}(t) S_n - R^{(0)}(t) K_n]. \end{aligned}$$

При подстановке полученных значений сумм имеем

$$\begin{aligned} p_t'(x) = -\rho [C_0'(t) + 2a C_0(t)] - \rho [C_1'(t) + 2a C_1(t)] x - \\ - \rho E(x) + \rho \frac{2}{l} [H^{(0)}(t) S_n + R^{(0)}(t) K_n] = \\ = -\rho \left[C_0'(t) + 2a C_0(t) - \frac{2}{l} [H^{(0)}(t) S_n + R^{(0)}(t) K_n] \right] - \\ - \rho [C_1'(t) + 2a C_1(t)] x + \rho E(x). \end{aligned}$$

Теперь интегрируем по x :

$$p_t(x) = -\rho \left\{ C_0'(t) + 2aC_0(t) - \frac{2}{l} \left[H^{(0)}(t)S_n + R^{(0)}(t)K_n \right] \right\} x - \rho \left[C_1'(t) + 2aC_1(t) \right] \frac{x^2}{2} + \rho \int_0^x E(\eta) d\eta.$$

Вставим полученное выражение в формулу для давления:

$$p(x, t) = \rho c^2 C_n^{(0)}(t) - \rho \left\{ C_0'(t) + 2aC_0(t) - \frac{2}{l} \left[H^{(0)}(t)S_n + R^{(0)}(t)K_n \right] \right\} x - \rho \left[C_1'(t) + 2aC_1(t) \right] \frac{x^2}{2} + \rho c^2 \sum_{n=1}^{\infty} \lambda_n \left[Y_n^{(0)}(t) + L_n^{(0)}(t) \right] \cos(\lambda_n x + \varphi_n) + \rho \int_0^x E(\eta) d\eta.$$

Значение последнего интеграла определяется согласно начальному распределению давления по длине участка, которое вытекает из первого уравнения системы (1):

$$p(x, 0) = p_{00} - \rho \int_0^x \left(w^{(t)}(\eta) + 2aw^{(0)}(\eta) \right) d\eta.$$

При $t=0$ члены, выделенные верхним индексом «0» и заглавными буквами, превращаются в нуль. В связи с этим для определения значения интеграла получим равенство:

$$p_{00} - \rho \int_0^x \left(w^{(t)}(\eta) + 2aw^{(0)}(\eta) \right) d\eta = -\rho \left[C_0'(t) + 2aC_0(t) \right] x - \rho \left[C_1'(t) + 2aC_1(t) \right] \frac{x^2}{2} + \rho \int_0^x E(\eta) d\eta.$$

Отсюда находим значение интеграла $\int_0^x E(\eta) d\eta$ и вставим в формулу для давления:

$$p(x, t) = p_{00} - \rho \int_0^x \left(w^{(t)}(\eta) + 2aw^{(0)}(\eta) \right) d\eta - \rho c^2 \left\{ C_1^{(0)}(t) + \sum_{n=1}^{\infty} \lambda_n \left[Y_n^{(0)}(t) + L_n^{(0)}(t) \right] \cos \lambda_n x \right\}.$$

$$\text{Здесь } C_1^{(0)}(t) = \frac{1}{l} \int_0^t \left[\bar{w}(\theta) - \tilde{w}(\theta) \right] d\theta.$$

Таким образом, выясняется, что

граничные значения скорости $\bar{w}(t)$, $\tilde{w}(t)$ – должны быть дважды дифференцируемые и интегрируемые функции, а выражения для $w^{(0)}(x)$ и $w^{(t)}(x)$ – интегрируемые функции.

Мы демонстрировали формальный ход решения задачи. В процессе решения задачи относительно $w(x, t)$ и $p(x, t)$ отдельные производные и интегралы считали уже известными. Но при расчетах ограничиваемся только рассмотрением задач о переходных процессах. Для каждого из случаев будем приводить конкретные значения производных и интегралов.

Результаты и их обсуждение

В варианте I расчет проводили при $w_{00} = 0$ м/с, $w^{(0)} = w^{(1)} = 5$ м/с. Т.е. в покое участке под высоким давлением 7.0 МПа на входе начало поступать жидкость со скоростью 5 м/с, а из выхода начали отбирать жидкость со скоростью 5 м/с. Масса жидкости на участке – постоянная.

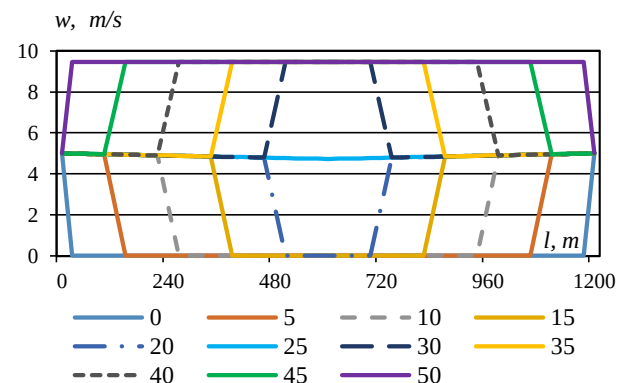


Рис. 1. Графики скорости по длине участка, полученные в первых десяти шагах времени с интервалом $l/(10c)$. $l=1200$ м, $D=0.2$ м,

$$\lambda=0.018, w_* = 5 \text{ м/с}, c=1200 \text{ м/с},$$

$$w_{00} = 0 \text{ м/с}, w^{(0)} = w^{(1)} = 5 \text{ м/с},$$

$$p_{00} = 7.00 \text{ МПа}$$

На рис. 1 представлены кривые скорости в различных шагах времени первого условного периода.

В данном варианте условный период процесса относительно показателей равен l/c . Нижние кривые (при $tc/(10l)=0, 5, 10, 15, 20$) относятся первой половине условного периода. Процесс возрастания скорости начинается

одновременно из двух концов участка: от нуля до 5 м/с . Волны постепенно перемешаются встречно. При $ct/(10l)=25$ они взаимно встречаются, образуя почти прямой линии с постоянной скорости 5 м/с . При этом приближение к середине участка происходит с потерей определенной скорости. Складывавшись этих импульсы, доводят скорость до нового значения, которая уже меньше чем $2 \cdot 5 \text{ м/с} = 10 \text{ м/с}$. После этого волны уплотнения и разряжения продолжают свои движения до достижения концов участка. Но опять происходит снижение значений скачка скорости на определенное значение.

Достигнув концы участка, волны переворачиваются и стремятся обратно к взаимной встрече. В отличие от первых четырех нижних графиков, здесь в нижних звеньях скорость имеет уже не нулевое, а приблизительно единичное значение. При возвращении волн повторяется процесс, но с меньшим наибольшим значением. И т.д. В нечетных условных периодах преобразуется верхняя огибающая линия кривых давления, а в четных – нижняя огибающая. Это суждение подтверждается также графиками рис. 2, которые представляют результатов изменения скорости по времени для сечений $x=24, 600, 912$ и 1176 м . Здесь можно наблюдать за характером постепенного перехода к режиму с постоянной скоростью 5 м/с .

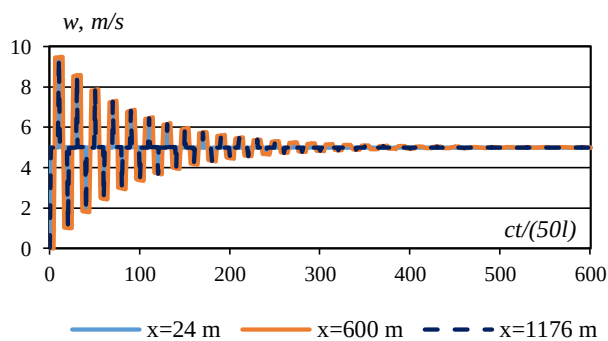


Рис. 2. Графики скорости потока в различных сечениях участка. Данные см. рис. 1

При использовании более крупного интервала времени (рис. 3) проявляется экспоненциальное убывание амплитуды давления. При этом предельные значения давления в концах и середине участка согласуются новыми значениями скорости

потока. В середине участка установилось значение, которое было принято в начале процесса.

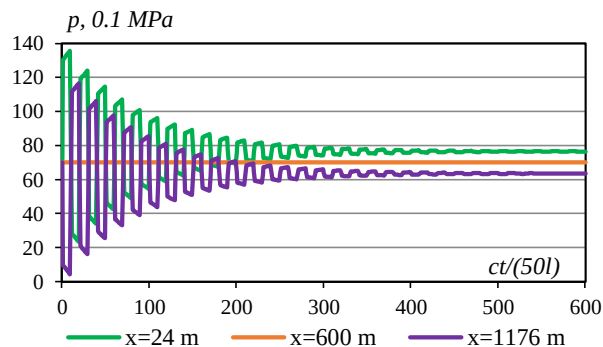


Рис. 3. Изменение давления по времени в различных сечениях трубопровода. Данные см. рис. 1

Во всех вариантах относительно скорости потока получены периодические решения. При $w^{(0)} \neq w^{(1)}$ получили решения с периодом $2c/l$, а при $w^{(0)} = w^{(1)}$ – с периодом c/l . В случае $a \neq 0$ эти величины составляли условные период, выражающие характер изменения показателей в участке. При этом ступеньки в графиках скорости, отделяемые скачками скорости и граничными условиями, получились строго горизонтальными. Т.е. интервал изменения скорости при скачках остается одинаковым при распространении односторонних возмущений. При встрече волн уплотнения и разряжения, образованных одинаковыми граничными скоростями, наблюдается мгновенная взаимная компенсация и опрокидывания волн, которые продолжают распространяться дальше без потерь (рис. 4).

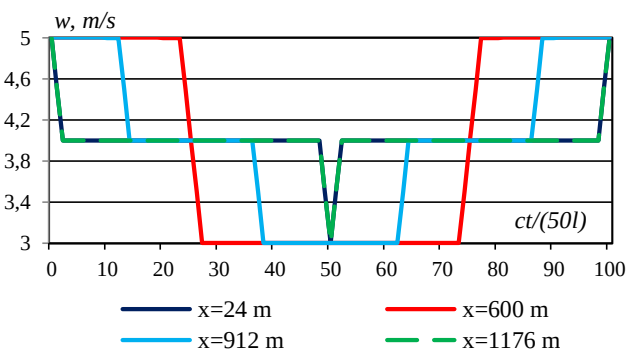


Рис. 4. Графики скорости по времени в разных сечениях участка при $a=0$. $w_{00} = 5 \text{ м/с}$,

$$w^{(0)} = w^{(1)} = 4 \text{ м/с}$$

Как видно из рисунка, при переходе к новому значению скорости с обеих сторон

участка интервал изменения скорости строго составляет . Амплитуда изменений скорости при односторонних возмущениях находится в интервале с длиной. Типичные для такого случая графики приведены рис. 5.

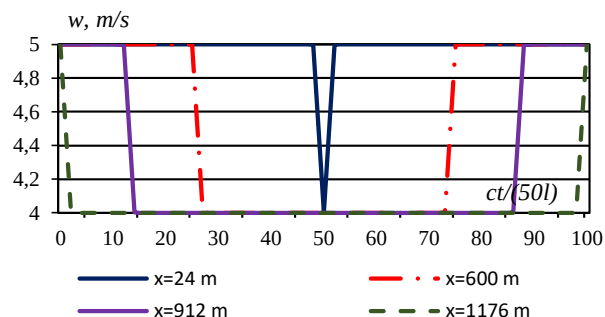


Рис. 5. Изменения скорости потока по времени в одном периоде в различных сечениях. $a = 0$,

$$w_{00} = w^{(0)} = 5 \text{ м/с}, w^{(1)} = 4 \text{ м/с}$$

Постоянство значений в ступеньках графиков остается в силе и для результатов расчета по давлению. Периодичность решения по давлению наблюдается при При происходит повсеместное возрастание давления в конце каждого периода (рис. 6).

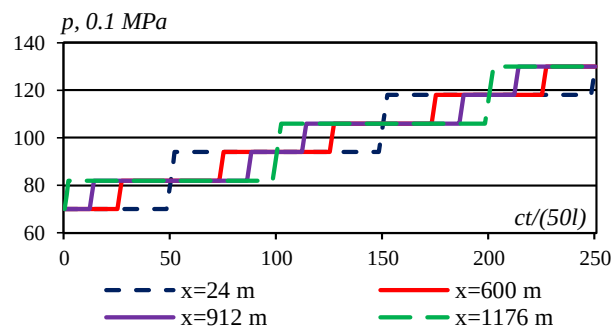


Рис. 6. Повсеместное возрастание давления при

$$w_{00} = w^{(0)} = 5 \text{ м/с}, w^{(1)} = 4 \text{ м/с}$$

При наблюдается противоположный эффект. Но в обоих случаях при исключении члена со множителем получается одинаковое решение для отдельно взятых периодов.

ЗАКЛЮЧЕНИЕ

Решение задачи по скорости потока получено методом Фурье в виде функционального ряда, а решение по давлению получено путем интегрирования исходных уравнений при известном решении по скорости.

Для случаев, когда функции в краевых условиях имеют постоянные значения, составлена программа расчета и проведен

вычислительный эксперимент с учетом и без учета силы сопротивления трения.

Изучены варианты переходных процессов в условиях линеаризованного квадратичного закона сопротивления, когда масса жидкости на участке постоянная, линейно возрастающая и линейно убывающая в зависимости от времени. Выявлены особенности распространения и отражения одно- и двухсторонних возмущений скорости в элементарном участке трубопровода, стремящиеся перестроить поля скорости и давления согласно новым граничным условиям с образованием своеобразных огибающих кривых.

С привлечением найденного значения скорости и исходных уравнений получено решение задачи для полного напора, который в случае горизонтального трубопровода переходит обычному давлению. В случае положительной разнице между входной и выходной скоростями ожидается монотонное возрастание давления в участке, а при отрицательной разнице – монотонное убывание. Скорость изменения среднего давления в элементарном участке пропорциональна величине разности между входной скоростью. В связи с этим при решении задачи имеет смысл только при малых отрезках времени.

Результаты работы получены для развитого турбулентного режима течения и круглого сечения трубопровода. Для применения результатов при расчете трубопровода с другой формой поперечного сечения вместо диаметра можно использовать удвоенный эквивалентный радиус трубопровода.

Статья выполнена за счет бюджетного финансирования Института механики и сейсмостойкости сооружений им. М.Т.Уразбаева АН РУз.

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Информация об авторах

1. **Хужаев Исматулла Кушаевич** – Д.т.н., профессор. Институт механики и сейсмостойкости сооружений им. М.Т. Уразбаева, Ташкент, Узбекистан. e-mail: ismatillo.khujayev@mail.ru

2. **Хожикулов Шахзод Шавкатжон ўгли** – базовый докторант. Институт механики и сейсмостойкости сооружений им. М.Т. Уразбаева, Ташкент, Узбекистан. e-mail: hojikulovshaxzod@mail.ru

3. **Махкамов Мадаминжон Комилович** – кандидат технических наук, доцент. Анджижанский государственный университет, проректор. e-mail: mailuzbek@umail.uz

4. **Нуманжанов Абдурахмон Абдурасул ўгли** – заместитель декана физика-математического и ИТ факультета

NEW COMBINATORIAL IDENTITIES BASED ON THE DISTRIBUTION OF CODEWORDS OVER HYPERCUBE FACES

J. K. Abdurakhmanov

Andijan State University, 170100, Republic of Uzbekistan, Andijan city, st. Universitet, 129

E-mail: abduraxmanov@adu.uz

2025 yil 2-iyulda berilgan; 2025 yil 1-noyabrda chop etish uchun qabul qiligan

Аннотация. Мы представляем новую структуру для изучения комбинаторных тождеств через распределений кодовых слов по граням гиперкуба. Мы доказываем, что для любого подмножества двоичного гиперкуба функция ранга удовлетворяет уточненным границам, которые приводят к точным вычислениям для малых мощностей. Наша основная теорема устанавливает тождества, связывающие число граней заданной размерности, содержащих ровно заданное число элементов подмножества, с биномиальными суммами по всем подмножествам указанной мощности. На основе этого мы открываем новые комбинаторные тождества. Этот геометрический подход выявляет скрытые симметрии в классических комбинаторных структурах и обеспечивает единую структуру для генерации новых комбинаторных тождеств.

Ключевые слова: Гиперкуб, грани гиперкуба, подмножества гиперкуба, распределение по граням, корректирующий код, комбинаторные тождества, расстояние Хэмминга, биномиальные коэффициенты, дискретное метрическое пространство, конечные множества, комбинаторика, дискретные множества.

Abstract. We present a new framework for studying combinatorial identities via distributions of codewords over hypercube faces. We prove that for any subset of the binary hypercube, the rank function satisfies refined bounds that lead to exact computations for small cardinalities. Our main theorem establishes identities relating the number of faces of a given dimension containing exactly a given number of elements of a subset to the binomial sums over all subsets of a given cardinality. Based on this, we discover new combinatorial identities. This geometric approach reveals hidden symmetries in classical combinatorial structures and provides a unified framework for generating new combinatorial identities.

Keywords: Hypercube, hypercube faces, hypercube subsets, face distribution, error-correcting code, combinatorial identities, Hamming distance, binomial coefficients, discrete metric space, finite sets, combinatorics, discrete sets.

Annotatsiya. Biz kod so'zlarining giperkub qirralari bo'yicha taqsimlanishini o'rganish orqali kombinator tēg'risida yangi struktura taklif qilamiz. Biz isbotlaymizki, ikkilik giperkubning istalgan kichik to'plami uchun rang funksiyasi aniqlashtirilgan chegaralarga bo'ysunadi va bu chegaralar kichik quvvatlar uchun aniq hisob-kitoblarga olib keladi. Asosiy teoremlar berilgan o'lchamdagi qirralar soni (ya'ni aynan ma'lum miqdordagi elementlarni o'z ichiga olgan qirralar soni) bilan shu quvvatga ega barcha kichik to'plamlar bo'yicha olinadigan binom yig'indilari o'rtasidagi tēg'risida bog'liqlikni o'rnatadi. Shu asosda biz yangi kombinator tēg'risida aniqlaymiz. Ushbu geometrik yondashuv klassik kombinator strukturalardagi yashirin simmetriyalarni ochib beradi va yangi kombinator tēg'risida hosil qilish uchun yagona uslubni ta'minlaydi.

Kalit so'zlar: giperkub, giperkub qirralari, giperkub kichik to'plamlari, qirralar bo'yicha taqsimlanish, tuzatuvchi kod, kombinator tēg'risida, Xemming masofasi, binomial koeffitsiyentlar, diskret metrik fazo, chekli to'plamlar, kombinatorika, diskret to'plamlar.

INTRODUCTION

The study of combinatorial identities has a rich history dating back to the fundamental works of Euler, Vandermonde, and Cauchy. In recent years, there has been renewed interest in discovering new identities through geometric and algebraic methods, as demonstrated in recent works on generalizations of classical identities [2] and modern approaches using complex analysis [3]. This paper introduces a novel geometric framework for generating combinatorial

identities by analyzing how subsets of q -valued cubes are distributed among the various dimensional faces of the cube. Our approach builds upon the concept of subset rank introduced in our previous work [1], which measures the dimension of the smallest face containing a given subset. By studying the distribution of subset elements across faces of different dimensions, we uncover a rich class of combinatorial identities that encode geometric properties of the

underlying space. The main contributions of this paper are threefold:

1. We establish refined bounds for the rank function in binary cubes that enable exact computation for small subsets.
2. We prove a general theorem relating face distributions to binomial sums, yielding a parametric family of identities.
3. We demonstrate that classical identities like Vandermonde's have natural geometric interpretations in our framework, and discover completely new identities.

Throughout this paper, we adopt the convention that binomial coefficients are defined as:

$$\binom{n}{k} = \begin{cases} \frac{n!}{k!(n-k)!}, & \text{if } k \geq 0, \\ 0, & \text{if } k < 0, \end{cases}$$

where n, k are integers and $n \geq 0$, with the standard convention $0! = 1$.

1. Preliminaries and Notation

We begin by recalling the basic definitions and results from [1] that form the foundation of our work.

Definition 1. The set of all n -dimensional vectors whose coordinates belong to $E_q = \{0, 1, \dots, q-1\}$ is called the n -dimensional q -valued cube and is denoted by E_q^n .

Definition 2. A k -dimensional face of E_q^n is the set of all vectors in E_q^n with some fixed $n-k$ coordinates.

The vectors of a subset $A \subset E_q^n$ are also called the vertices, points, or elements of A . A fundamental concept in our framework is the rank of a subset:

Definition 3. The rank of a subset

$A \subset E_q^n$, denoted $R(A)$, is the dimension of the smallest face containing A .

This is equivalent to the number of variable columns in the matrix M_A whose rows consist of all vectors in A .

Two subsets A and B of E_q^n are called isometric if there exists a bijective mapping $\varphi: A \rightarrow B$ preserving the Hamming distance:

$$d_H(a, a') = d_H(\varphi(a), \varphi(a'))$$

for all $a, a' \in A$. We denote by J_A the isometry class containing A .

For a subset A , we define D_A as the sum of Hamming distances between all distinct pairs of vectors in A :

$$D_A = \sum_{\substack{x, y \in A \\ x \neq y}} d_H(x, y).$$

2. Refined Bounds for Binary Cubes

Our first main result refines the bounds for $R(A)$ established in [1] for the special case of binary cubes.

Theorem 1. For any subset $A \subset E_2^n$ with $|A| \geq 1$ and any $B \in J_A$, the following relations hold:

$$1) \quad R(B) = 0, \text{ if } |A| = 1,$$

$$2) \quad \frac{4D_A}{|A|^2} \leq R(B) \leq \frac{D_A}{|A|-1}, \text{ if } |A| >$$

1 and $|A|$ is even,

$$3) \quad \frac{4D_A}{|A|^2-1} \leq R(B) \leq \frac{D_A}{|A|-1}, \text{ if } |A| >$$

1 and $|A|$ is odd.

Proof. The case $|A| = 1$ is trivial. For $|A| > 1$, since isometric subsets have the same rank and distance sum, it suffices to prove the bounds for A itself.

By Theorem 3.1 in [1], for $q = 2$ and $|A| \geq 2$, we have:

$$\frac{4D_A}{|A|^2} \leq R(A) \leq \frac{D_A}{|A|-1}.$$

For odd $|A|$, we need to prove the stronger lower bound. Consider the matrix M_A whose rows are the vectors of A . Let the first $R(A)$ columns be variable, with column j having x_j zeros and y_j ones, where $x_j + y_j = |A|$. Then:

$$D_A = \sum_{j=1}^{R(A)} x_j y_j.$$

When $|A|$ is odd, to maximize D_A , we must have

$$\{x_j, y_j\} = \{(|A|-1)/2, (|A|+1)/2\}$$

for each j . This gives:

$$D_A \leq R(A) \cdot \frac{|A|^2-1}{4},$$

which yields the desired lower bound. ■

This theorem allows exact computation of ranks for small subsets. For $A \subset E_2^n$:

• If $A = \{a_1\}$, then $R(A) = 0$,

• If $A = \{a_1, a_2\}$, then $R(A) = d_{12} = D_A$,

• If $A = \{a_1, a_2, a_3\}$, then $R(A) =$

$$\frac{d_{12}+d_{13}+d_{23}}{2} = \frac{D_A}{2},$$

where $d_{ij} = d_H(a_i, a_j)$.

3. Distribution of Elements in Faces

To state our main results on face distributions, we first recall the concept of multisets. A *multiset* on

a finite set U is the set U together with a multiplicity function $r: U \rightarrow N_0$. The cardinality of a multiset is $\sum_{a \in U} r(a)$. The sum of multisets $U_1, U_2, \dots, U_l$ with multiplicity functions $r_1, r_2, \dots, r_l$ is the multiset with multiplicity function $r(a) = r_1(a) + r_2(a) + \dots + r_l(a)$, denoted $\cup_{i=1}^l U_i$.

Let $v(A, k)$ denote the number of k -dimensional faces of E_q^n containing the subset A .

Lemma 1. For any non-empty subset $A \subset E_q^n$ and integer k with $0 \leq k \leq n$:

$$v(A, k) = \binom{n - R(A)}{k - R(A)}.$$

Proof. If $k < R(A)$, then $v(A, k) = 0$ and the formula holds. For $k \geq R(A)$, to specify a k -dimensional face containing A , we must include all $R(A)$ variable positions and choose $k - R(A)$ additional positions from the remaining $n - R(A)$ constant positions. The result follows. ■

Let $V(A, k, e)$ denote the number of k -dimensional faces containing exactly e elements of A .

Theorem 2. Given a non-empty subset $A \subset E_q^n$, for any integers k and s with $0 \leq k \leq n, 1 \leq s \leq p(k)$, where $p(k) = \min\{|A|, q^k\}$:

$$\sum_{e=s}^{p(k)} \binom{e}{s} V(A, k, e) = \sum_{B \subset A, |B|=s} \binom{n - R(B)}{k - R(B)},$$

where the right sum is over all s -element subsets $B \subset A$.

Proof. Let $\Gamma(B, k)$ be the set of all k -dimensional faces containing B . Consider the multiset:

$$\Gamma' = \cup_{B \subset A, |B|=s} \Gamma(B, k).$$

In Γ' , each k -dimensional face containing exactly e elements of A has multiplicity $\binom{e}{s}$, so:

$$|\Gamma'| = \sum_{e=s}^{p(k)} \binom{e}{s} V(A, k, e).$$

On the other hand:

$$\begin{aligned} |\Gamma'| &= |\cup_{B \subset A, |B|=s} \Gamma(B, k)| = \sum_{B \subset A, |B|=s} |\Gamma(B, k)| \\ &= \sum_{B \subset A, |B|=s} \binom{n - R(B)}{k - R(B)}, \end{aligned}$$

using Lemma 1. ■

4. Combinatorial Identities from Geometric Structures

Theorem 2 provides a general framework for deriving combinatorial identities. We now present several corollaries and examples.

Corollary 1. Let $A \subset E_q^n$, k be an integer with $0 \leq k \leq n$, and $p(k) = \min\{|A|, 2^k\}$. Then:

1) For $|A| \geq 1$:

$$\sum_{e=1}^{p(k)} \binom{e}{1} V(A, k, e) = \sum_{e=1}^{p(k)} e V(A, k, e) = |A| \binom{n}{k};$$

2) For $|A| \geq 2$:

$$\sum_{e=2}^{p(k)} \binom{e}{2} V(A, k, e) = \sum_{i=1}^{|A|-1} \sum_{j=i+1}^{|A|} \binom{n - d_{ij}}{k - d_{ij}};$$

3) For $|A| \geq 3$:

$$\sum_{e=3}^{p(k)} \binom{e}{3} V(A, k, e) = \sum_{i=1}^{|A|-2} \sum_{j=i+1}^{|A|-1} \sum_{t=j+1}^{|A|} \binom{n - d_{ijt}}{k - d_{ijt}}$$

where $d_{ijt} = \frac{d_{ij} + d_{it} + d_{jt}}{2}$ and d_{ij}, d_{it}, d_{jt} are the corresponding Hamming distances between pairs of vectors $a_i, a_j, a_t \in A$.

These identities remain valid for general $q > 2$ in cases 1) and 2).

4.1. Example: Vandermonde's Identity from Face Geometry

Let $A \subset E_q^n$ be a v -dimensional face. The values of $V(A, k, e)$ can be computed explicitly:

1) For $e = 0$:

$$V(A, k, e) = \sum_{i=0}^v q^{v-i} \binom{v}{i} \binom{n-v}{k-i} (q^{n-v-k+i} - 1);$$

2) For $e = q^i, i = 1, 2, \dots, v$:

$$V(A, k, e) = q^{v-i} \binom{v}{i} \binom{n-v}{k-i};$$

3) For other values of e :

$$V(A, k, e) = 0.$$

Substituting into **Corollary 1** part 1) with $|A| = q^v$:

$$\sum_{i=0}^v \binom{v}{i} \binom{n-v}{k-i} = \binom{n}{k},$$

which is Vandermonde's identity.

For part 2), the number of two-element subsets $B \subset A$ with $R(B) = i$ is

$$\frac{1}{2} (q-1)^i \binom{v}{i} q^v.$$

This leads to the new identity:

$$\begin{aligned} \sum_{i=1}^v (q^i - 1) \binom{v}{i} \binom{n-v}{k-i} \\ = \sum_{i=1}^v (q-1)^i \binom{v}{i} \binom{n-i}{k-i}. \end{aligned}$$

Note that this identity is true for any integers n, k, q, v that satisfy conditions $n \geq 1, 0 \leq k \leq n, 1 \leq v \leq n, q \geq 1$. This

generalizes results on Chu-Vandermonde variations studied by Mestrovic [2].

4.2. Example: Even-Weight Vectors

Let $A \subset E_2^n$ consist of all even-weight vectors (those with an even number of 1's). Then:

$$V(A, k, e) = \begin{cases} 2^{n-k} \binom{n}{k}, & \text{if } e = 2^{k-1}, \\ 0, & \text{if } e \neq 2^{k-1}. \end{cases}$$

From **Corollary 1** part 2):

$$(2^{k-1} - 1)2^{n-1} \binom{n}{k} = \sum_{i=1}^{\lfloor \frac{n}{2} \rfloor} \binom{n}{2i} \binom{n-2i}{k-2i}.$$

This is a completely new combinatorial identity valid for all integers $n, k, 1 \leq k \leq n$.

5. Applications and Connections

The geometric framework developed here has several important applications:

5.1. Linear Codes

When q is a prime power, E_q^n becomes a vector space over $GF(q)$. Linear codes are subspaces, and our results provide new tools for analyzing their weight distributions. The distinction between algebraic dimension and combinatorial rank offers insights into code structure. Recent work on graph covers [4] shows similar geometric approaches can yield unexpected results in combinatorial structures.

5.2. Matroid Theory

The rank function $R(A)$ defines a matroid on subsets of E_q^n . Our identities translate to statements about the characteristic polynomial and Tutte polynomial of these matroids. This connects to recent developments in matroid polynomials, such as the work by Ferroni and Larson [5] on Kazhdan-Lusztig polynomials of braid matroids.

5.3. Quantum Error Correction

Binary and q -ary quantum codes correspond to special subsets of E_q^n . Our framework provides new approaches to constructing codes with prescribed distance properties.

6. Conclusion

We have introduced a geometric framework for studying combinatorial identities through the distribution of subset elements in faces of q -valued cubes. This approach reveals that many classical identities have natural geometric interpretations and

enables the discovery of new identities. The connection to enumerative combinatorics, as systematically developed in Stanley's comprehensive treatment [6], suggests many further directions for exploration.

The main contributions include refined bounds for subset ranks in binary cubes, a general theorem connecting face distributions to binomial sums, and the discovery of new identities for even-weight vectors and generalizations of Vandermonde's identity.

Future directions include:

- a) Extending the framework to infinite-dimensional spaces;
- b) Developing algorithms for computing $V(A, k, e)$ for general subsets;
- c) Finding q -analogue versions of our identities;
- d) Applications to extremal combinatorics and coding theory.

The geometric perspective on combinatorial identities opens new avenues for research at the intersection of combinatorics, geometry, and algebra.

We note that the main results were previously reported in the preprint [7].

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Muallif haqida ma'lumot

Jamolidin Komoldinovich Abduraxmanov – fizika-matematika fanlari nomzodi, dotsent, Andijon davlat universiteti Dasturiy injiniring kafedrasi dotsenti.

E-mail: abduraxmanov@adu.uz
Telefon: +998 94 104 41 23

MASHINALI O'QITISH TEXNOLOGIYASIDAN FOYDALANIB PLASTINALARNING DEFORMATSIYALANISH JARAYONLARINI MODELLASHTIRISH

Mirzaaxmedov M.K, Ovixonov I.A.

¹Zahiriddin Muhammad Bobur Andijon davlat universiteti, Andijon, O'zbekiston

2025-yil 2-iyulda berilgan; 2025-yil 1-noyabrda chop etish uchun qabul qilingan

Аннотация. В данной статье рассматриваются возможности моделирования процессов деформации тонких пластин с использованием технологий машинного обучения. Разработана модель на основе искусственных нейронных сетей для определения нелинейных зависимостей между температурой, напряжением и деформационными параметрами пластин. В ходе исследования в среде программирования Python были протестированы алгоритмы машинного обучения (линейная регрессия, случайный лес и нейронная сеть) для анализа деформационных состояний пластин на основе экспериментальных данных. Результаты показывают, что нейронная сеть обеспечивает точность на 18–25% выше по сравнению с традиционными аналитическими моделями. Данный подход открывает новые возможности для прогнозирования параметров прочности и упругости технических материалов.

Ключевые слова. Машинное обучение, пластина, деформация, моделирование, нейронная сеть, эластичность, прогнозирование.

Abstract. In this work explores the possibilities of modeling the deformation processes of thin plates using machine learning technologies. In the article, a model based on artificial neural networks was developed to determine the nonlinear relationships between the temperature, stress, and deformation parameters of plates. During the study, machine learning algorithms (Linear Regression, Random Forest, and Neural Network) were tested in the Python programming environment to analyze the deformation states of plates based on experimental data. The results show that the neural network model provided 18–25% higher accuracy compared to traditional analytical models. This approach creates new opportunities for predicting the strength and elasticity parameters of technical materials.

Key words. Machine learning, plate, deformation, modeling, neural network, elasticity, prediction.

Annotatsiya. Ushbu maqolada mashinali o'qitish (machine learning) texnologiyalaridan foydalanib yupqa plastinalarning deformatsiyalanish jarayonlarini modellashtirish imkoniyatlari o'rganildi. Maqolada plastinalarning harorat, kuchlanish va deformatsiya parametrlari o'rtasidagi nelinear bog'lanishlarni aniqlash uchun sun'iy neyron tarmoqlarga asoslangan model ishlab chiqildi. Tadqiqot davomida plastinalarning deformatsiya holatlarini eksperimental ma'lumotlar asosida tahlil qiluvchi Python dasturiy muhitida mashinali o'qitish algoritmlari (Linear Regression, Random Forest va Neural Network) sinovdan o'tkazildi. Natijalar ko'rsatishicha, neyron tarmoq modeli an'anaviy analitik modellar bilan solishtirganda 18–25% yuqori aniqlikni ta'minladi. Ushbu yondashuv texnik materiallarning mustahkamlik va elastiklik parametrlarini bashorat qilishda yangi imkoniyatlar yaratadi.

Kalit so'zlar. Mashinali o'qitish, plastina, deformatsiya, modellashtirish, neyron tarmoq, elastiklik, bashoratlash.

KIRISH

Zamonaviy sanoat va muhandislikda plastinalarning deformatsiyalanish jarayonlarini aniq modellashtirish masalasi muhim o'rin tutadi. Mexanik yuklama, harorat, va tashqi kuchlar ta'sirida yuz beruvchi deformatsiyalarni oldindan aniqlash, qurilmalar ishonchligini ta'minlashda asosiy omil hisoblanadi.

An'anaviy analitik modellar, xususan, elastiklik nazariyasiga asoslangan differensial tenglamalar, ko'p hollarda soddalashtirishlar va taxminlarga tayanadi. Bu esa murakkab geometrik shaklli plastinalar uchun aniqlikning pasayishiga olib keladi. Shu sababli, so'nggi yillarda **mashinali o'qitish texnologiyalari** asosida fizik jarayonlarni bashoratlash va modellashtirishga qiziqish ortib bormoqda.

Mashinali o'qitish yondashuvi katta hajmdagi eksperimental yoki simulatsion ma'lumotlardan o'rganib, nolinear bog'lanishlarni avtomatik tarzda aniqlay oladi. Bu yondashuv materiallarning xatti-

harakatini har xil yuklama, harorat va deformatsiya sharoitlarida aniqroq tahlil qilish imkonini beradi.

Maqolaning asosiy maqsadi — mashinali o'qitish modellaridan foydalanib yupqa plastinalarning deformatsiyalanish jarayonlarini modellashtirish va an'anaviy analitik yondashuvlar bilan taqqoslab, ularning samaradorligini baholashdir.

ADABIYOTLAR TAHLILI

Ushbu maqola chuqur o'rganish (deep learning) nazariyasining asoslarini, xususan, neyron tarmoqlar, optimallashtirish usullari va o'qitish jarayonlarini batafsil tushuntiradi [1]. Plastinalarning deformatsiya jarayonini modellashtirishda qo'llaniladigan sun'iy neyron tarmoqlar tuzilmasini yaratishda nazariy asos bo'lib xizmat qiladi. Tadqiqot ishida modellashtirish jarayonida qo'llanilgan neyron tarmoqlarning arxitekturasini tanlashda metodologik asos sifatida ishlatilgan. Mazkur kitob mashinali o'qitish algoritmlarining nazariy asoslari, xususan regressiya,

bayes tahlili va ehtimoliy modellarni qamrab oladi. Deformatsiya jarayonida kuzatilgan nolinear bog'lanishlarni aniqlashda mashinali o'qitishning matematik tamoyillari muhimdir. Plastinalarning deformatsiyasi jarayonida harorat, kuch va deformatsiya o'rtasidagi bog'lanishni regressiya modellarida ifodalash uchun asos bo'lib xizmat qiladi [2]. Maqola materialshunoslikda mashinali o'qitish yondashuvlarining qo'llanilishi — mexanik, issiqlik va struktura xossalarini bashoratlash bo'yicha keng tahlil beradi [3]. Mashinali o'qitish yordamida materiallarning mustahkamligi va deformatsion xatti-harakatini oldindan baholash mumkinligini isbotlaydi. Tadqiqot plastinalarning fizik deformatsiyasini tahlil qilishda mashinali o'qitish usullarining samaradorligini asoslaydi. Plastinalar va qobiq elementlarining elastiklik nazariyasi, deformatsiya va kuchlanish hisoblari keltirilgan klassik asar. Plastinalarning fizik xatti-harakatini an'anaviy matematik model orqali tushuntiradi [4]. Tadqiqotda mashinali o'qitish yondashuvlari natijalari ushbu an'anaviy model bilan solishtiriladi. Metall deformatsiyasi jarayonini sun'iy neyron tarmog'i yordamida modellashtirish tajribasi bayon etilgan. Neyron tarmoqlar real tajriba ma'lumotlaridan o'rganib, deformatsiyani aniq bashorat qilish imkonini beradi [5]. Ushbu ish maqoladagi plastina deformatsiyasi uchun neyron tarmoq modelini loyihalashda amaliy manba sifatida xizmat qilgan. Mashinali o'qitish algoritmlaridan foydalangan holda materiallarning elastiklik modulini bashoratlash usullari o'rganilgan [6]. Deformatsiyani aniqlashda elastiklik modulining aniqligi juda muhim bo'lib, mashinali o'qitish bu parametрни tez va ishonchli hisoblash imkonini beradi.

Maqoladagi deformatsiya modeli uchun material elastik parametrlarini o'rganishda foydalanilgan. Chetki harorat yuklamalari ostidagi yupqa plastinalarda termik stresslarni chuqur neyron tarmoqlar yordamida tahlil qilish ko'rib chiqilgan. Termo-deformatsion modellashtirishda mashinali o'qitishning qo'llanishi ko'rsatilgan. Mazkur maqola plastina deformatsiyasiga harorat ta'sirini neyron tarmoqlar yordamida modellashtirishga bevosita asos bo'lgan [7]

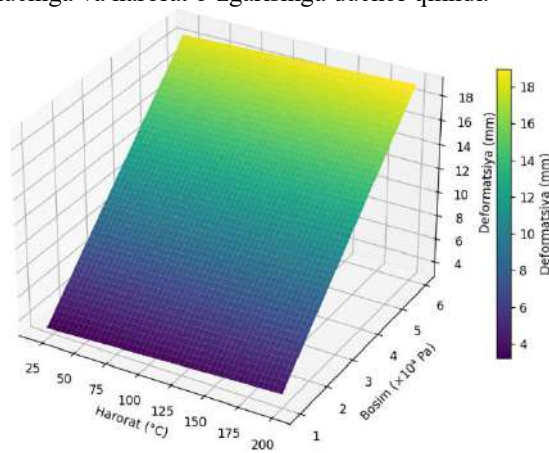
Adabiyotlar tahlili shuni ko'rsatadiki, mashinali o'qitish va sun'iy neyron tarmoqlari yordamida deformatsion jarayonlarni modellashtirish hozirgi kunda faol rivojlanayotgan ilmiy yo'nalishlardan biridir. Tahlil qilingan manbalar quyidagi ilmiy xulosalarni tasdiqlaydi:

- Mashinali o'qitish material xossalarini bashoratlashda samarali.
- Neyron tarmoqlar nolinear fizik bog'lanishlarni aniqlashda an'anaviy modellardan ustun.
- Fizik qonunlarni ma'lumotlar asosida modellashtirish (PINN) deformatsion tahlilni yangi bosqichga olib chiqmoqda.

MATERIAL VA USULLAR

Tadqiqot obyekti sifatida alyuminiy qotishmasidan tayyorlangan $200 \times 200 \times 2$ mm o'lchamdagi yupqa

plastina tanlandi. Plastina markaziy qismidan bosim kuchiga va harorat o'zgarishiga duchor qilindi.



1-rasm. Alyuminiy plastina deformatsiyasi (3D ko'rinish)

Eksperimental ma'lumotlar harorat (T), tashqi kuch (F), va plastina deformatsiyasi (δ) ko'rsatkichlarini o'z ichiga oldi.

1-jadval

№	T (°C)	F (N)	δ (mm)
1	25	50	0.12
2	35	55	0.15
...	...	...	...

500 ta o'lchov natijalari asosida mashinali o'qitish modeli tayyorlandi.

2-jadval

Model	Tavsif	Amalga oshirish muhiti
Linear Regression	Deformatsiyani harorat va kuchlanishga chiziqli bog'lash uchun	scikit-learn
Random Forest	Nolinear bog'lanishlarni aniqlash uchun	scikit-learn
Neyron tarmoq (ANN)	3 qatlamli model orqali bashoratlash uchun	TensorFlow

Model baholash mezonlari

Model aniqligi quyidagi mezonlar asosida baholandi:

1. O'rtacha kvadrat xato (MSE)

O'rtacha kvadrat xato (MSE) — bu model tomonidan bashorat qilingan deformatsiya qiymatlari bilan haqiqiy (tajriba yo'li bilan o'lchangan) qiymatlar orasidagi o'rtacha kvadrat farqni o'lchovchi aniqlik mezon.

$$MSE = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2 \quad (1)$$

y_i — plastinaning **haqiqiy deformatsiya qiymati**;

$\hat{y}_i$ — model orqali **bashorat qilingan**

deformatsiya qiymati;

n — namunalar soni.

2. Determinatsiya koeffitsienti (R^2)

Deformatsiya koeffitsienti (belgilanadi: ε , “epsilon”) — bu jismlarning kuch ta'sirida uzunligi, yuzasi yoki hajmi o'zgarish miqdorining dastlabki o'lchamga nisbati. Ya'ni u nisbiy o'zgarish (koeffitsient) bo'lib, o'lchovsiz kattalik hisoblanadi.

$$\varepsilon = \frac{\Delta L}{L_0} \quad (2)$$

ε — deformatsiya koeffitsienti (nisbiy deformatsiya),

$\Delta L = L - L_0$ — uzunlikdagi o'zgarish (mm yoki m),

L_0 — dastlabki uzunlik (mm yoki m)

3. O'rtacha absolyut xato (MAE)

O'rtacha absolyut xato (MAE) — bu bashorat qilingan qiymatlar bilan haqiqiy (tajriba) qiymatlar orasidagi farqlarning absolyut qiymati o'rtachasi. Ya'ni, MAE modelning o'rtacha xato darajasini ko'rsatadi, lekin har bir xatoning yo'nalishini (musbat yoki manfiy) hisobga olmaydi.

$$MAE = \frac{1}{n} \sum_{i=1}^n |y_i - \hat{y}_i| \quad (3)$$

y_i — **haqiqiy deformatsiya qiymati**;

$\hat{y}_i$ — model orqali **bashorat qilingan qiymati**;

$|y_i - \hat{y}_i|$ — model tomonidan bashorat qilingan qiymat,

n — o'lchovlar soni (namunalar soni).

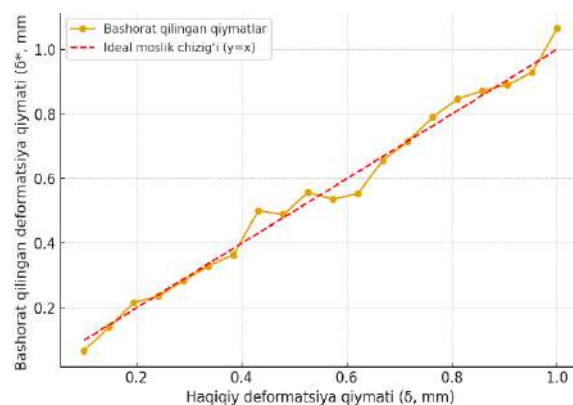
NATIJALAR

Natijalarga ko'ra, mashinali o'qitish modellari orasida **1. sun'iy neyron tarmog'i** eng yuqori aniqlikni ko'rsatdi.

3-jadval

Model	MSE	MAE	R ²
Linear Regression	0.0045	0.051	0.87
Random Forest	0.0022	0.034	0.93

Neural Network	0.0015	0.028	0.95
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1-rasm. Bashoratlangan va haqiqiy deformatsiya qiymatlarining taqqoslash grafigi

MUHOKAMA

Olingan natijalar shuni ko'rsatadiki, mashinali o'qitish texnologiyalari, ayniqsa **sun'iy neyron tarmoqlari**, murakkab fizik jarayonlarni modellashtirishda samarali vosita bo'la oladi.

An'anaviy analitik modellar ko'pincha deformatsiya–kuch–harorat bog'lanishlarini chiziqli deb hisoblaydi, bu esa real sharoitlarda xatolikni oshiradi. Mashinali o'qitish yondashuvi esa ma'lumotlardan o'zi mustaqil tarzda o'rganadi va nolinear munosabatlarni avtomatik aniqlaydi [8].

Bundan tashqari, Random Forest modeli ham barqaror natijalar berdi, biroq u neyron tarmoq darajasida aniqlikka erishmadi. Shu bilan birga, Random Forest modeli kamroq o'qitish vaqti talab qiladi, bu uni tezkor tahlillar uchun qulay qiladi.

Amaliy natijalar shuni ko'rsatadiki, ushbu metodologiya ishlab chiqarish jarayonlarida plastinalarning deformatsion holatini tezda baholash va nazorat qilish tizimlariga integratsiya qilinishi mumkin.

XULOSA

Tadqiqot natijalari quyidagi xulosalarga olib keldi:

- Mashinali o'qitish texnologiyalari yordamida plastinalarning deformatsiyalanish jarayonlarini modellashtirish mumkin va bu usul an'anaviy modellarga qaraganda yuqori aniqlik beradi.
- Neyron tarmoq asosidagi model deformatsiya parametrlarini 95% aniqlik bilan bashorat qildi.
- Harorat va kuchlanish parametrlari deformatsiyaga nolinear ta'sir ko'rsatadi, bu mashinali o'qitish algoritmlari yordamida samarali aniqlanadi.
- Taklif etilgan yondashuv ishlab chiqarishdagi plastinalar sinovini avtomatlashtirish va optimallashtirishda qo'llanilishi mumkin.

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Mualliflar haqida ma'lumot

Muhammadbobur Karimberdiyevich Mirzaaxmedov

Zahiriddin Muhammad Bobur nomidagi Andijon davlat universiteti Kompyuter injiniringi kafedrasi dotsenti (PhD). 170100, Andijon viloyati, Andijon shahar, Universitet ko'chasi 129-uy.
e-mail: muhammadbobur1110@mail.ru.

Iqboljon Abdunabiyevich Ovxunov

Zahiriddin Muhammad Bobur nomidagi Andijon davlat universiteti, Kompyuter injiniringi kafedrasi mudiri, dotsent, (PhD) 170100, Andijon viloyati, Andijon shahar, Universitet ko'chasi 129-uy
e-mail: iqboljon_ovxunov@gmail.com

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2) фамилия, имя и отчество автора (-ов);

3) краткая аннотация на узбекском и русском языках (450–500 знаков с пробелами), аннотация выделяется курсивом и отделяется от текста статьи пропуском строки;

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2. Текст статьи набирается шрифтами Times New Roman, размер шрифта – 14 кеглей, межстрочный интервал – одинарный, поля – сверху и снизу 2 см, слева 3 см, справа 1,5 см; абзацный отступ – 1,25 см. с электронной версией (на флешке или компакт-диске). В названии файла указываются фамилия и специальность автора, например: *Физика_Бакиров*.

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5) английский вариант списка литературы (References). В списке сначала дается транслитерация ФИО автора, названия источника и его выходных данных. Далее в квадратных скобках перевод названия источника на английский язык. References должен быть оформлен в соответствии со стандартом АПА-2010.

